Euclidean wormholes, baby universes, and their impact on particle physics and cosmology

Arthur Hebecker (Heidelberg)

including original work with
P. Mangat / S. Theisen / L. Witkowski and P. Soler / T. Mikhail
and T. Daus / J. March-Russell / S. Leonhardt

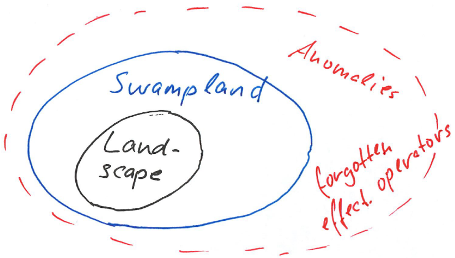
Outline

- Instantons and the Weak Gravity Conjecture for axions
- Euclidean wormholes as the 'WGC-objects'
- 'Coleman's wormholes' and their problems
- Recent developments: Black hole entropy, Global Symmetries

Key questions leading to 'The Swampland':

Vafa '05, Ooguri/Vafa '06

- Does 'anything go' in the landscape of consistent gravitational 4d EFTs / string-compactifications ?
- Are there general criteria for a given model not to be in the landscape?
- Can we formulate and prove such citeria in 'consistent quantum gravity' (rather than specifically in string theory)?



Two ‘Swampland Criteria’ important for this talk:

No exact global symmetries

see e.g. Banks/Dixon ’88, Kamionkowksi/March-Russell, Holman et al. ’92, Kallosh/Linde^2/Susskind ’95, Banks/Seiberg ’10, Harlow/Ooguri ’18

The weak gravity conjecture

Arkani-Hamed/Motl/Nicolis/Vafa ’06

- Roughly speaking: ‘Gravity is always the weakest force.’
- More concretely:
  For any U(1) gauge theory there exists a charged particle with

$$\boxed{m < q} \qquad \text{(with } q = gn \text{ and } M_P = 1\text{)}\,.$$

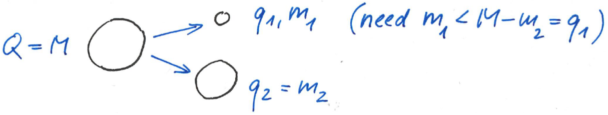
Weak gravity conjecture (continued)

- The historical supporting argument:

  In the absence of sufficiently light, charged particles, extremal BHs are stable. Such remnants are believed to cause inconsistencies.

  see e.g. Susskind '95

  Indeed, the boundary of stability of extremal black holes is precisely $q/m = 1$ for the decay products.

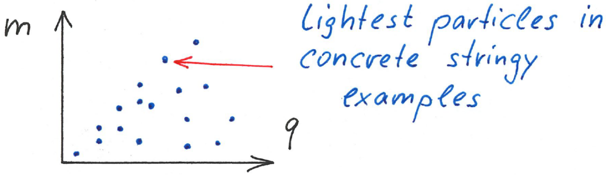


Weak gravity conjecture (continued)

- Another (possibly stronger?) supporting argument:

  Quantum gravity forbids global symmetries. We should not be able to take the limit of small gauge couplings.

  The WGC quantifies this on the basis of stringy examples.



- It is not obvious how the WGC could impact phenomenology.
- Interesting proposals have been made by

  Ooguri/Vafa, Palti, Ibanez/Valenzuela/Martin-Lozano/
  Montero, Reece, ... '16...'19.

- One of the widely accepted applications is to constraining large-field inflation by constrainig axions

  Cheung/Remmen; de la Fuente/Saraswat/Sundrum . . . '14
  Rudelius; Ibanez/Montero/Uranga/Valenzuela; Brown/Cottrell/Shiu/Soler/..
  ..Staessens/Ye; Bachlechner/Long/McAllister; AH/Rompineve/Witkowski;
  Junghans; Heidenreich/Reece/Rudelius; Kooner/Parameswaran/Zavala;
  Harlow; AH/Rompineve/Westphal; . . . '15
  Ooguri/Vafa, Conlon/Krippendorf . . . '16
  Dolan/Draper/Kozaczuk/Patel; AH/Henkenjohann/Witkowski/Soler . . . '17

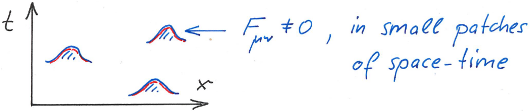
- The connection to axions will lead us to wormholes in just a few slides....

Axions

- Both in pheno-model-building and in string compactifications, axion-like fields are abundant:

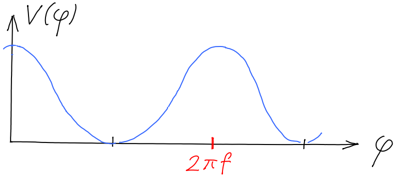
$$\mathcal{L} \supset -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{32\pi^2}\left(\frac{\varphi}{f}\right)\mathrm{tr}(F\tilde{F})\,.$$

- Their shift symmetry is generically broken by instantons:



$$\Rightarrow \quad V_{eff} \sim \cos(\varphi/f)\,,$$

$$\varphi \equiv \varphi + 2\pi f\,.$$



...to apply it to axions, the previously discussed WGC needs to be generalized:

Generalizations of the weak gravity conjecture

- The basic lagrangian underlying the above is

$$S \sim \int (F_2)^2 + m \int_{1-dim.} d\ell + q \int_{1-dim.} A_1 \,.$$

- This generalizes to charged strings, domain walls etc. Crucially, the degree of the corresponding form-field (gauge-field) changes:

$$S \sim \int (F_{p+1})^2 + m \int_{p-dim.} dV + q \int_{p-dim.} A_p$$

with

$$F_{p+1} = dA_p \,.$$

Generalizations to instantons

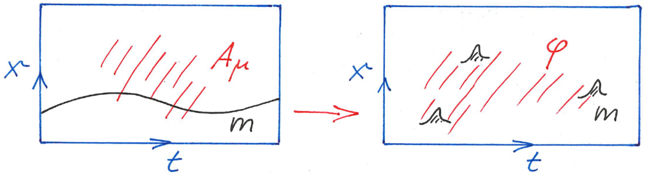
- One can also lower the dimension of the charged object, making it a point a in space-time:

$$S \sim \int (d\varphi)^2 + m + q\,\varphi(x_{inst.})\,.$$

This should be compared with

$$\text{cf.} \quad S \sim \int (d\varphi)^2 + \int \mathrm{tr}(F^2) + \int \frac{1}{f}\,\varphi\,\mathrm{tr}(F\tilde{F})\,,$$

$$\text{where} \quad \int \mathrm{tr}(F^2) \sim S_{inst.} \sim m\,.$$



WGC for instantons and axions

- The consequences for inflation are easy to derive.
- First, recall that the instantons induce a potential

$$V(\varphi) \sim e^{-S_{inst.}} \cos(\varphi/f)\,.$$

- Since, for instantons, $q \to 1/f$ and $m \to S_{inst.}$ we have

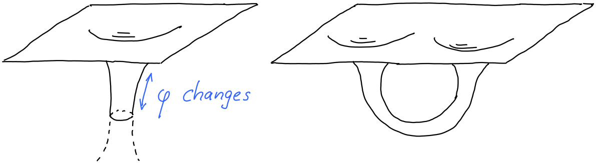
$$m < q \qquad \Rightarrow \qquad \boxed{S_{inst.} < 1/f}\,.$$

- Theoretical control (dilute instanton gas) requires $S_{inst.} > 1\,.$
- This implies $f < 1$ and hence
large-field 'natural' inflation is in trouble.
- Moreover, even for $f < 1$ one gets a lower bound on the
strength of instanton effects: $\exp(-S_{inst.}) > exp(-1/f)$.

A distinct but WGC-related tool: (Gravitational) instantons

- In Euclidean Einstein gravity, supplemented with an axionic scalar φ , instantonic solutions exist:

Giddings/Strominger '88
...



- The 'throat' is supported by the kinetic energy of $\varphi = \varphi(r)$, with r the radial coordinate of the throat/instanton.

- The relevance for inflation arises through the induced instanton-potential for the originally shift-symmetric field φ.

Montero/Uranga/Valenzuela '15

Gravitational instantons (continued)

- The underlying lagrangian is simply

$$\mathcal{L} \sim \mathcal{R} + f^2 |d\varphi|^2 \,, \quad \text{now with} \quad \varphi \equiv \varphi + 2\pi \,.$$

- This can be dualized $(dB_2 \equiv f^2 * d\varphi)$ to give

$$\mathcal{L} \sim \mathcal{R} + \frac{1}{f^2} |dB_2|^2 \,.$$

- The 'throat' exists due the compensation of these two terms. Reinstating M_P, allowing n units of flux (of $H_3 = dB_2$) on the transverse S^3, and calling the typical radius R, we have

$$M_P^2 \, R^{-2} \sim \frac{n^2}{f^2} R^{-6} \quad \Rightarrow \quad M_P \, R^2 \sim \frac{n}{f} \,.$$

Gravitational instantons (continued)

- Returning to units with $M_P = 1$, their instanton action is

$$S \sim n/f \qquad \text{(with } n \text{ the instanton number).}$$

- Very intriguingly, this coincides parametrically with the lowest-action instanton of the WGC.

- The maximal WH-curvature scale is $\sqrt{f/n}$, which should not exceed the UV cutoff:

$$f/n < \Lambda^2$$

- This fixes the lowest n that we can trust and hence the minimal size of the instanton correction to the potential $V(\varphi)$:

$$\boxed{\delta V \sim e^{-S} \sim e^{-n/f} \sim e^{-1/\Lambda^2}}$$

Gravitational instantons (continued)

- For gravitational instantons not to prevent inflation, the relative correction must remain small:

$$\frac{\delta V}{V} \sim \frac{e^{-1/\Lambda^2}}{H^2} \ll 1$$

- For a Planck-scale cutoff, $\Lambda \sim 1$, this is never possible
- However, the UV cutoff can in principle be as low as H
- Then, if also $H \ll 1$, everything might be fine....

$$\frac{\delta V}{V} \sim \frac{e^{-1/H^2}}{H^2}$$

AH, Mangat, Rompineve, Witkowski '15

For more details see e.g. Heidenreich/Reece/Rudelius '15, AH/Mangat/Theisen/Witkowski '16, Hertog/Trigiante/Van Riet '17, ... Andriolo/Huang/Noumi/Ooguri/Shiu '20 ...

...However, beyond inflation, wormholes remain very interesting, both conceptually and phenomenologically

Gravitational instantons - Small-f axions

Coleman/Lee, Rey $\sim$'90 .... Alonso/Urbano '17 .... Alvey/Escudero '20

- For example, for a QCD axion with (relatively) high f, the wormhole effect might be relevant:

$$V(\varphi) = \Lambda_{QCD}^4 \cos(\varphi) + r_c^{-4} e^{-S_w/2} \cos(\varphi + \delta)\,.$$

- It turns out that for $f \gtrsim 10^{16}$ GeV the solution to the strong CP problem is lost.
- Interesting positive observational consequences exist in the context of black-hole superradiance and ultralight dark matter.

Example: Fuzzy Dark Matter

Alonso/Urbano '17
AH/Mikhail/Soler '18

- Fuzzy Dark Matter is, by definition, so light that its de Broglie wavelength affects sub-galactic-scale structure: $m \lesssim 10^{-21}$ eV
- If wormholes are the universal, model-independent effect of shift symmetry breaking, then this fixes f by the relation

$$m^2 \sim \exp(-1/f)$$

- At the same time, the abundance of Fuzzy Dark Matter is given by

$$\Omega_{FDM} h^2 \approx 0.1 \left(\frac{f}{10^{17}\text{GeV}} \right)^2 \left(\frac{m}{10^{-22}\text{eV}} \right)^{\frac{1}{2}}$$

- Together, these two relations lead to a slight clash (between wormhole/WGC and Fuzzy Dark Matter pheno):

One finds $\quad m \gtrsim 10^{-19}\,\text{eV}\,, \quad$ ...slightly too high...

- Clearly, there are ways around this...

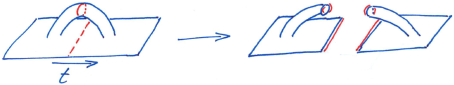
Gravitational instantons / wormholes - conceptual issues

- Motivated by the above, it is worthwhile revisiting some very fundamental conceptual issues of (euclidean) wormholes.

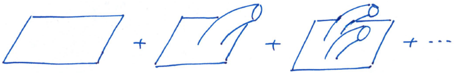
  Hawking '78..'88, Coleman '88, Preskill '89
  Giddings/Strominger/Lee/Klebanov/Susskind/Rubakov/Kaplunovsky/..
  Fischler/Susskind/...

  Recent review: AH, P. Soler, T. Mikhail '18

- First, once one allows for wormholes, one has to allow for baby universes.

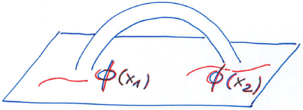


- Second, with baby universes comes a 'baby universe state' (α vacuum) encoding information on top of our 4d geometry.



Conceptual issues (continued)

- Crucially, α-parameters remove the disastrous-looking bilocal interaction.



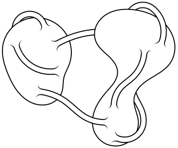
$$\exp\left(\int_{x_1}\int_{x_2}\Phi(x_1)\Phi(x_2)\right) \quad \to \quad \int_\alpha \exp\left(-\frac{1}{2}\alpha^2+\alpha\int_x \Phi(x)\right)$$

- In our concrete (single-axion) case, an α parameter now governs the naively calculable $e^{-S}\cos(\varphi/f)$-term.

- But, what is worse, all coupling constants are 'renormalized' by α parameters are hence not predictable in principle.

Conceptual issues (continued)

- Most naively, 4d measurements collapse some of the many α parameters to known constants.
- But in a global perspective, both different 4d geomtries and α parameters have to be integrated over.
- But this leads to the 'Fischler-Susskind-Kaplunovsky catastrophy'.



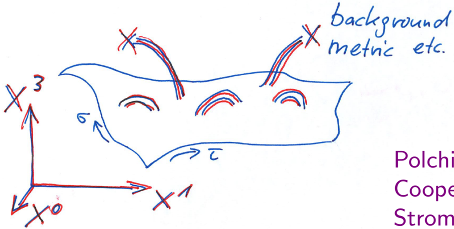
- Another key problem is a possible clash with locality on the CFT-side of AdS/CFT

Arkani-Hamed/Orgera/Polchinski '07, ..., 'SYK'

- Finally, just integrating over the α parameters is clearly not sufficient - one needs to consider their full quantum dynamics.

Conceptual issues (continued)

- Indeed, consider the case of 1+1 dimensions with a number of scalar fields (in addition to gravity).
- This is, of course, well known as string theory and the α parameters characterize the geometry the target space.



Polchinski, Banks/Lykken/O'Loughlin, Cooper/Susskind/Thorlacius, Strominger '89...'92

- The latter has a quantum dynamics of its own, the analogue of which in case of 3+1 dimensions is completely unknown.
- All this raises so many complicated issues, that one might want to dismiss wormholes altogether.

Conceptual issues (continued)

- But this is not easy, for example because we know that in string theory wormholes correspond to string loops and are a necessary part of the theory.

- Thus, forbidding for example topology change in general does not appear warranted.

- Is there a good reason to forbid topology change just in $d > 2$?

- Arguments have been given that the euclidean Giddings-Strominger solution has negative modes and should hence be dismissed.

  Rubakov/Shvedov '96, Maldacena/Maoz '04, see however Alonso/Urbano '17, ...

- But, while this is even technically still an open issue, it does not appear to be a strong enough objection ....

Conceptual issues (continued)

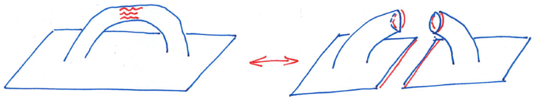
- Indeed, once a non-zero amplitude

  universe $\to$ universe $+$ baby-universe

  is accepted, the reverse process is hard to forbid.

- As a result, one gets all the wormhole effects.

- The negative mode issue may be saying:
  'Giddings-Strominger' does not approximate the amplitude well.



- ..hard to see, how it would dispose of the problem altogether..

For further problems (and possible resolutions) see e.g.
Bergshoeff/Collinucci/Gran/Roest/Vandoren/Van Riet '04,
Arkani-Hamed/Orgera/Polchinski '07, Hertog/Trigiante/Van Riet '17

Recent development: Wormholes and BH entropy

(very briefly)

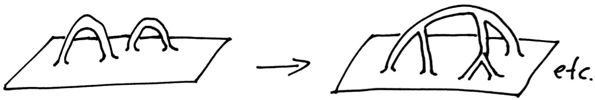
- Recently, a concrete proposal for calculating the entropy of an evaporating BH has emerged (method of 'Islands')

Penington, Almheiri/Engelhardt/Marolf/Maxfield, Almheiri/Mahajan/Maldacena/Zhao, ... '19/20

- The concrete mechanism by which entropy leaves the BH in this approach is related to euclidean WHs

- Motivated by this, a new 2d toy model developing Coleman's baby universe calculation has been suggested

Marolf/Maxfield '20



Recent development: Wormholes and BH entropy (continued)

- In particular, Marolf/Maxfield proposed to mod out the naive BU Hilbert space by a certain equivalence (related to 1 BU $\rightarrow$ 2 BU transitions, etc.)

- It has then be proposed that, in $d \geq 4$, this equivalence should be so strong that the BU Hilbert space is 1-dimensional

  McNamara/Vafa '20

- This would not remove the effect of BUs completely, but it would get rid of the arbitrariness of α parameters

- But can we do a proper calculation in $d \geq 4$?

Recent development: Global Symmetry Conjecture

(based on recent work with Daus/Leonhardt/March-Russell)

- Axion = scalar with gauged discrete shift symmetry (by $2\pi n$)
- Wormhole/instanton effects break its continuous shift symmetry very weakly (non-perturbatively)
- Natural question: Can this be used to apply the Weak Gravity / Swampland logic to quantitatively constrain global symmetry violation? (of lin.-realized global symm.s)
- A first attempt has been made by invoking the familiar BH evaporation effect: Fichet/Saraswat '20

  Claim: In a thermal plasma, the BH-induced violation effect should not exceed the effect of symmetry-violating local operators.

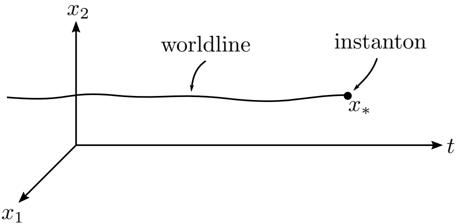
- The above BH argument is simply a new conjecture.
- By contrast, we claim that for an important sub-class (gauge-derived global symmetries) an actual derivation of a bound from the WGC is possible:

  Daus/AH/March-Russell/Leonhardt '20
- **Gauge-derived global symmetry means:**

  Gauge an axion with a $U(1)$ vector field;

  The leftover in the IR are the light $U(1)$-charged states, but now only protected by a global symmetry.
- Instantons automatically destroy such globally-charged particles

  (cf. many stringy examples)



Recent development: Global Symmetry Conjecture (continued)

- Thus, by the WGC for axions the particle-number violation is suppressed by

$$\exp(-S_{inst.}) \ \sim \ \exp(-M_P/f)$$

- Moreover, according to the magnetic WGC for axions there is a UV-cutoff due to light strings:

AH/Soler '17

$$\Lambda \ \sim \ \sqrt{M_P f}$$

Hence, in total the global-symmetry violation is bounded below by

$$\exp(-S_{inst.}) \ \sim \ \exp(-M_P^2/\Lambda^2)$$

- Very intriguingly, this is the same as the plasma-derived bound of Fichet/Saraswat and as the bound expected from wormholes:

$$S_{WH} \ \sim \ M_P^2 \int \mathcal{R} \ \sim \ M_P^2/\Lambda^2 \, .$$

Summary/Conclusions

- The WGC for axions demands certain minimal-action instantons and hence certain minimal potentials.
- Euclidean WHs may be the universal, semiclassical counterpart of WGC-instantons.
- They do not constrain inflation strongly, but may have other phenomenological applications 'at small f'.
- They come at the price of α vacua (and other disasters).
- Keep struggling with these fundamental unresolved issues!

- Recent result: WG provides universal bound $> \exp(-1/\Lambda^2)$ for the breaking of linearly-realized global symmetries.