# Quantum limited spin transport in ultracold atomic gases

## Searching for the perfect SPIN fluid...

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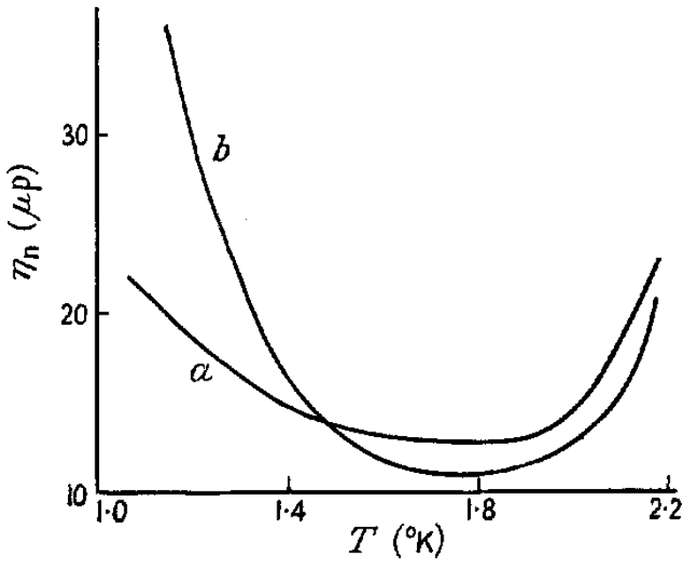
Enss and Haussmann, PRL **109**, 195303 (2012)

# Introduction

- Is ideal fluid realized in Nature? shear viscosity η=0?

helium-4:

[Heikkila, Hollis-Hallett 1955]




- find minimum: universal bounds on transport coefficients?

- not *all* coefficients bounded: e.g., bulk viscosity ζ=0 in scale-invariant fluids, but η is not protected by any symmetry

# Estimating the shear viscosity

- shear viscosity η on vastly different scales: normalize by entropy density s,

$$\frac{\eta}{s} = \# \frac{\hbar}{k_B}$$

ħ: indicates quantum effect

- kinetic theory (non-relativistic, 3D): $\eta \approx \frac{1}{3} n\, p\, \ell_{\rm mfp}\,, \qquad s \simeq k_B n$

  Fermi momentum $p \simeq \hbar k_F$

  mean free path $\ell_{\rm mfp} = 1/(n\sigma) \simeq \frac{1}{k_F}$ with cross section $\sigma \simeq \frac{1}{k_F^2}$ (unitarity)

- holographic duality (AdS/CFT): string theory dual to gravitational theory

  η: absorption cross section of black hole ~ event horizon

  s: entropy of black hole ~ event horizon

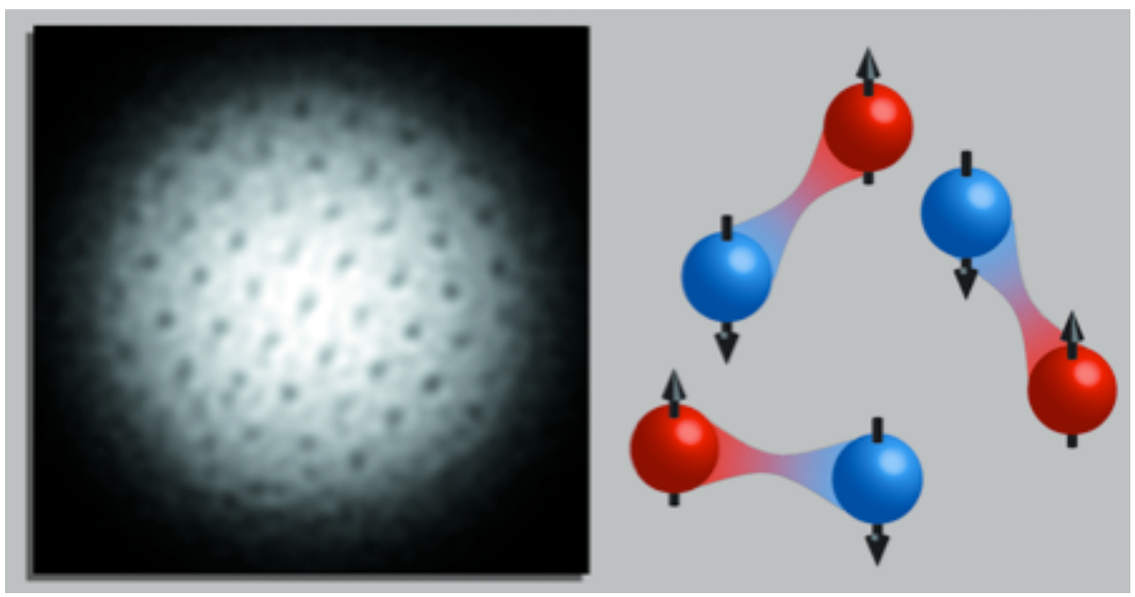
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

**perfect fluidity** [Kovtun, Son, Starinets 2005]

# Unitary Fermi gas

[Bloch, Dalibard, Zwerger 2008]

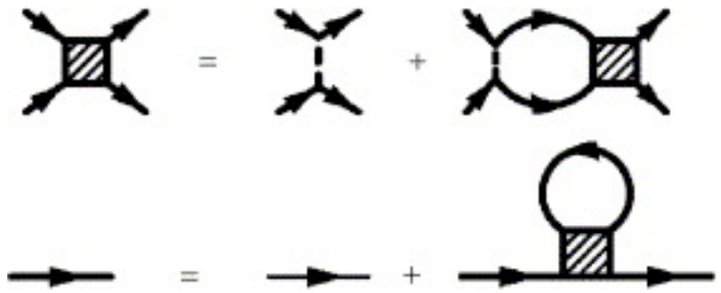
- non-relativistic Fermi gas, two spin components ↑, ↓
- contact interaction between ↑ and ↓: $|r_0| \ll \ell$
- strong s-wave scattering, $|a| \gg \ell$ (Feshbach resonance)
- superfluid of fermion pairs below $T_c/T_F \approx 0.16$ [Ku et al. Science 2012]



[Ketterle 2005]

# Theory for η/s

- 2PI (Luttinger-Ward) computation: repeated particle-particle scattering

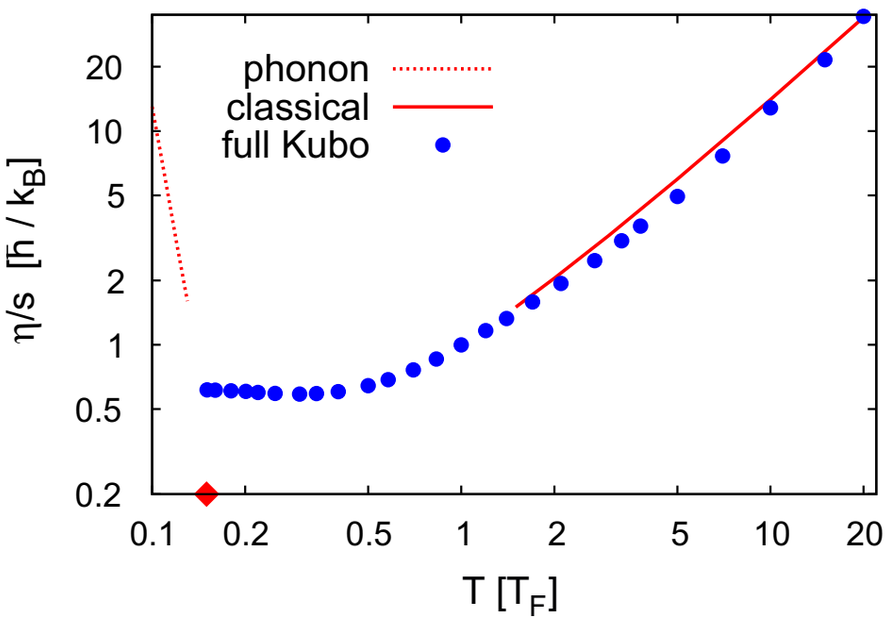


self-consistent T-matrix (300 mom./300 freq.)

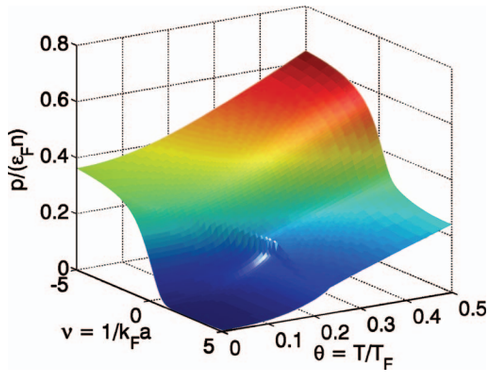
good approximation for unitary Fermi gas:

$$T_c/T_F \simeq 0.16, \quad \xi \simeq 0.36$$ [Haussmann et al. 2007]

- transport calculation via Kubo formula:



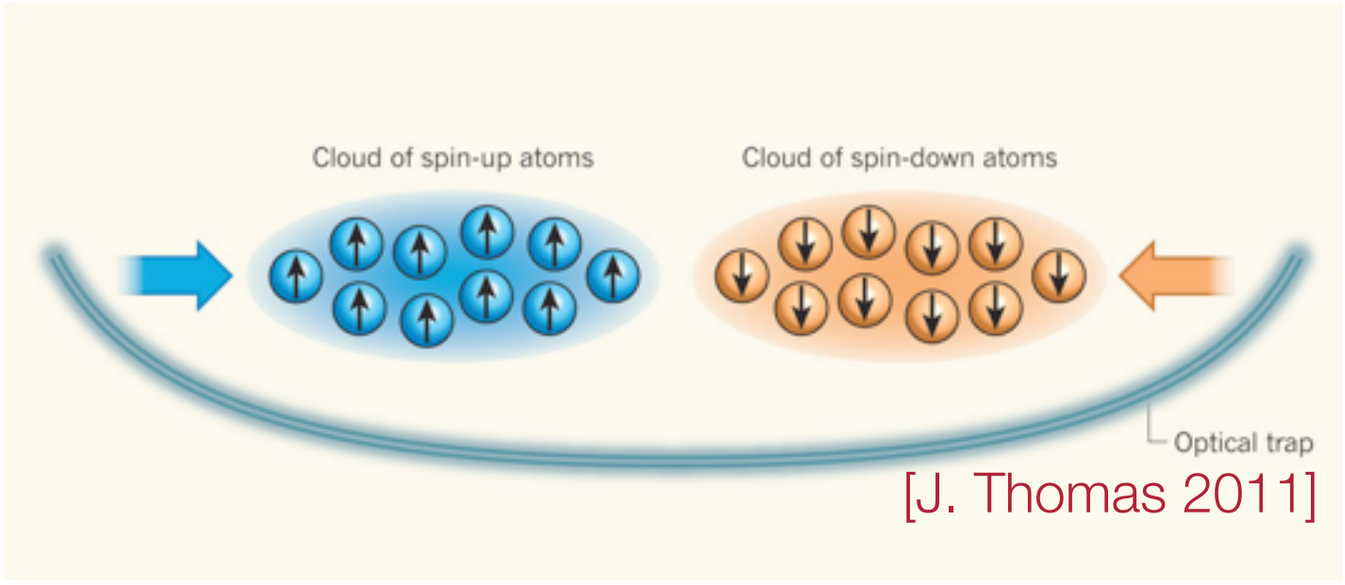

[Enss, Haussmann, Zwerger 2011]




[see also
Bruun, Smith 2007 (kin),
Enss 2012 (large-N),
Wlazlowski+ 2012 (QMC),
Schäfer, Chafin 2012 (hyd),
Romatschke, Young 2012]

# How about **spin** transport?

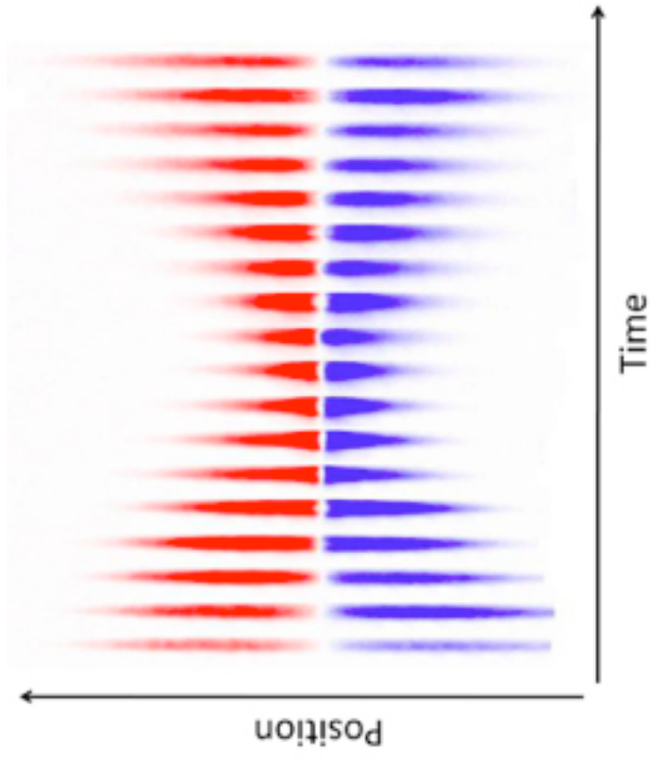
- experiment: spin-polarized clouds in harmonic trap




[J. Thomas 2011]

**bounce!**

- strongly interacting gas [movie courtesy Martin Zwierlein]:

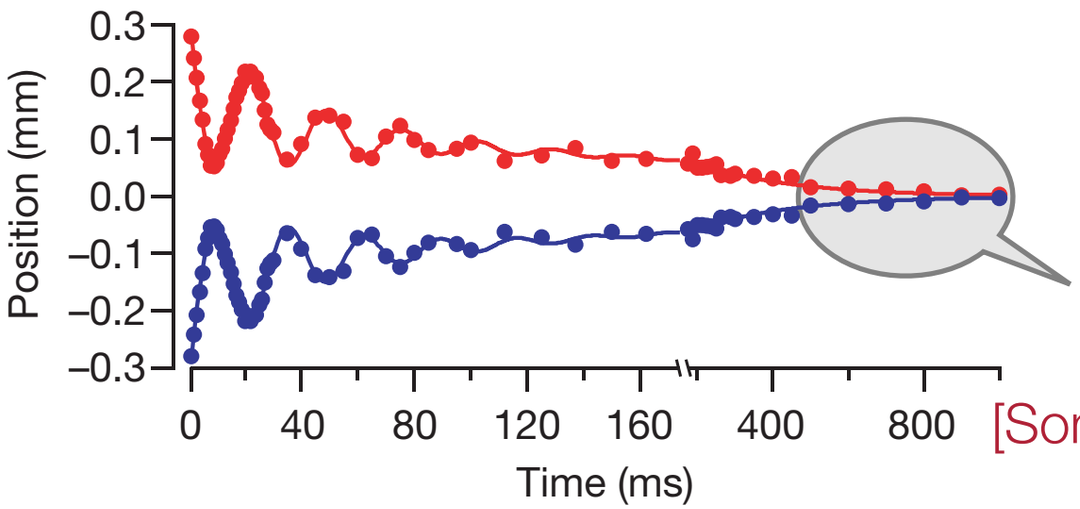



[A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)]

# Is there a quantum bound for spin diffusion?

- scattering conserves total ↑+↓ momentum: mass current preserved

  but changes relative ↑-↓ momentum: spin current decays




**spin diffusion**

[Sommer et al. 2011]

- **kinetic theory:** diffusion coefficient $D_s \approx v\ell_{\mathrm{mfp}}$ [Bruun 2011]

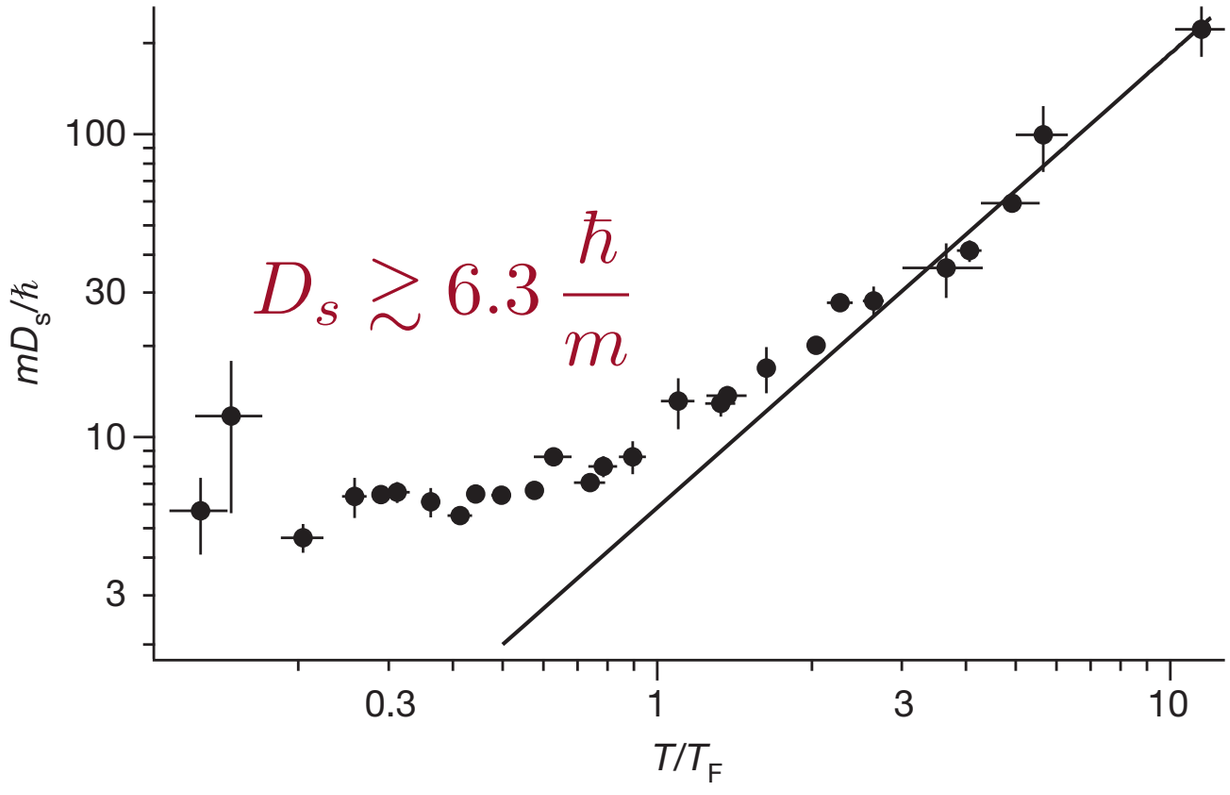
  Fermi velocity $v \simeq \dfrac{\hbar k_F}{m}$

  mean free path $\ell_{\mathrm{mfp}} = \dfrac{1}{n\sigma} \simeq \dfrac{1}{k_F}$ with cross section $\sigma \simeq \dfrac{1}{k_F^2}$ (unitarity)

  $\Longrightarrow \quad \boldsymbol{D_s \simeq \dfrac{\hbar}{m}}$ **quantum limit for diffusion**

# Spin diffusivity

- cold atom experiment: $D_s = \dfrac{\text{area}}{\text{time}} \approx \dfrac{(100\,\mu\text{m})^2}{(1\,\text{second})} \approx \dfrac{\hbar}{m}$




[Sommer et al. 2011]

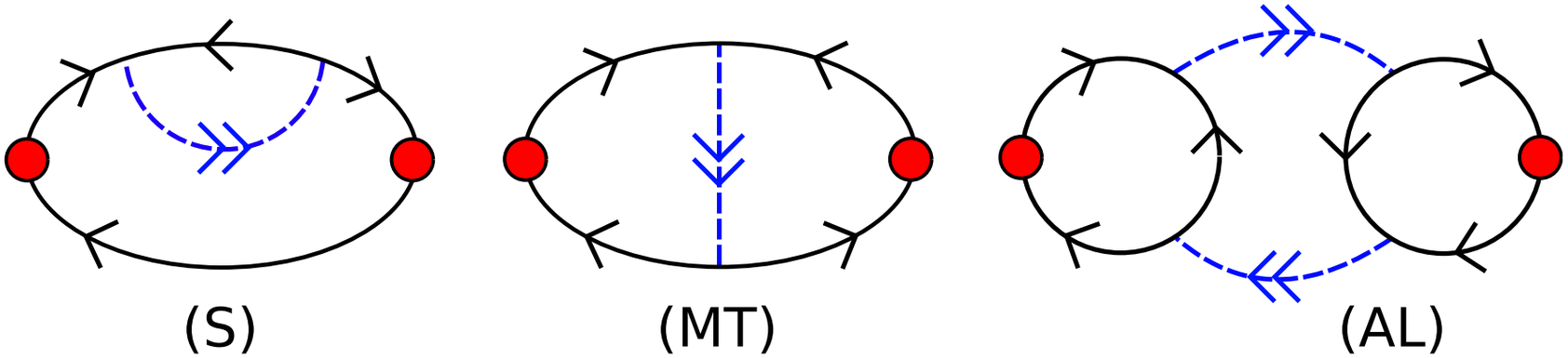
- solid state: spin Coulomb drag in GaAs quantum wells $D_s \simeq 500\,\dfrac{\hbar}{m}$ [Weber 2005]

# Computing the spin diffusivity

- 2PI (Luttinger-Ward) theory: use Einstein relation $D_s = \dfrac{\sigma_s}{\chi_s}$

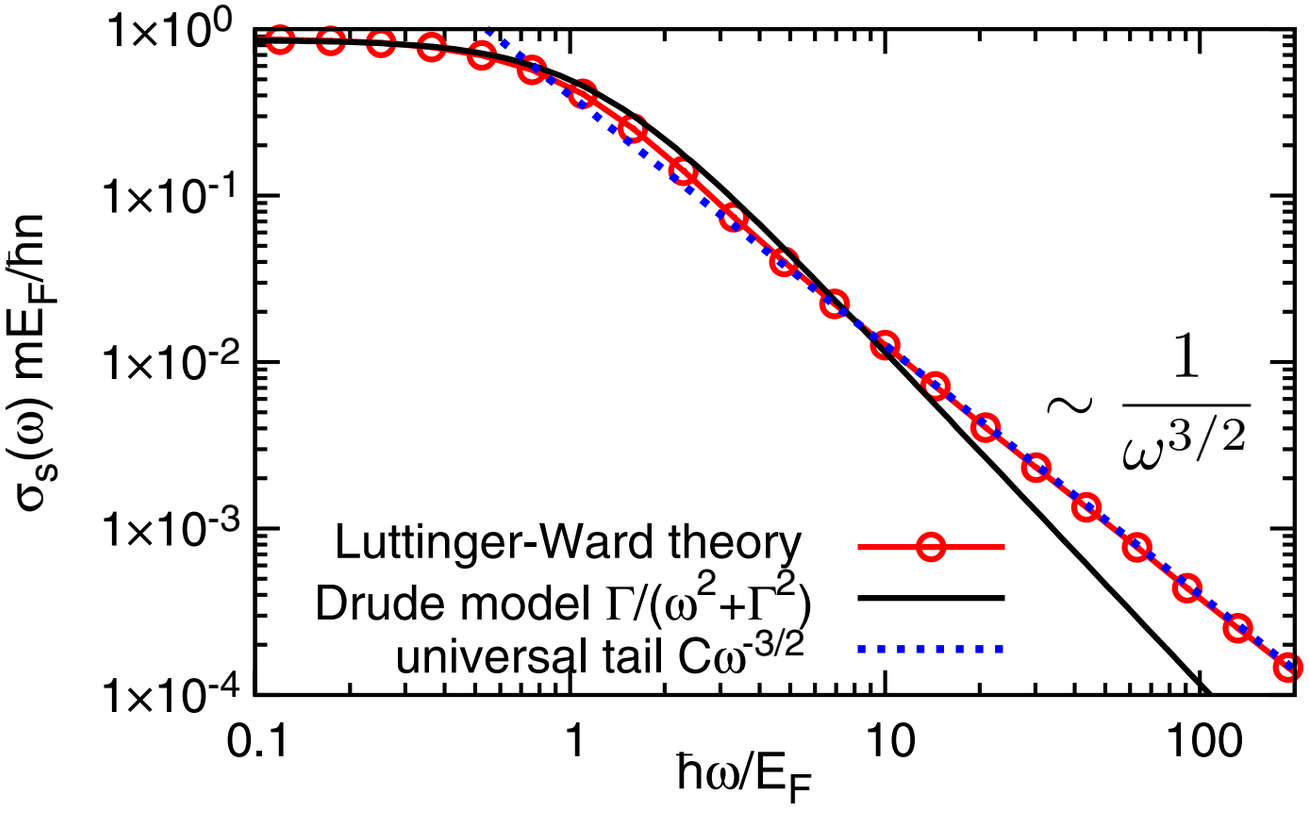
  spin conductivity $\sigma_s(\boldsymbol{q}, \omega)$ from current correlation fct. $\langle [j_\uparrow - j_\downarrow,\, j_\uparrow - j_\downarrow] \rangle$

  - including vertex corrections to satisfy ⬆, ⬇ particle number conservation




- including medium effects [Enss, Küppersbusch, Fritz 2012]

# Dynamical spin conductivity




- **exact** high-frequency tail
  [Enss, Haussmann PRL 2012]

$$\sigma_s(\omega \to \infty) = \frac{C}{3\pi(m\omega)^{3/2}}$$

C: contact density [Tan 2008]

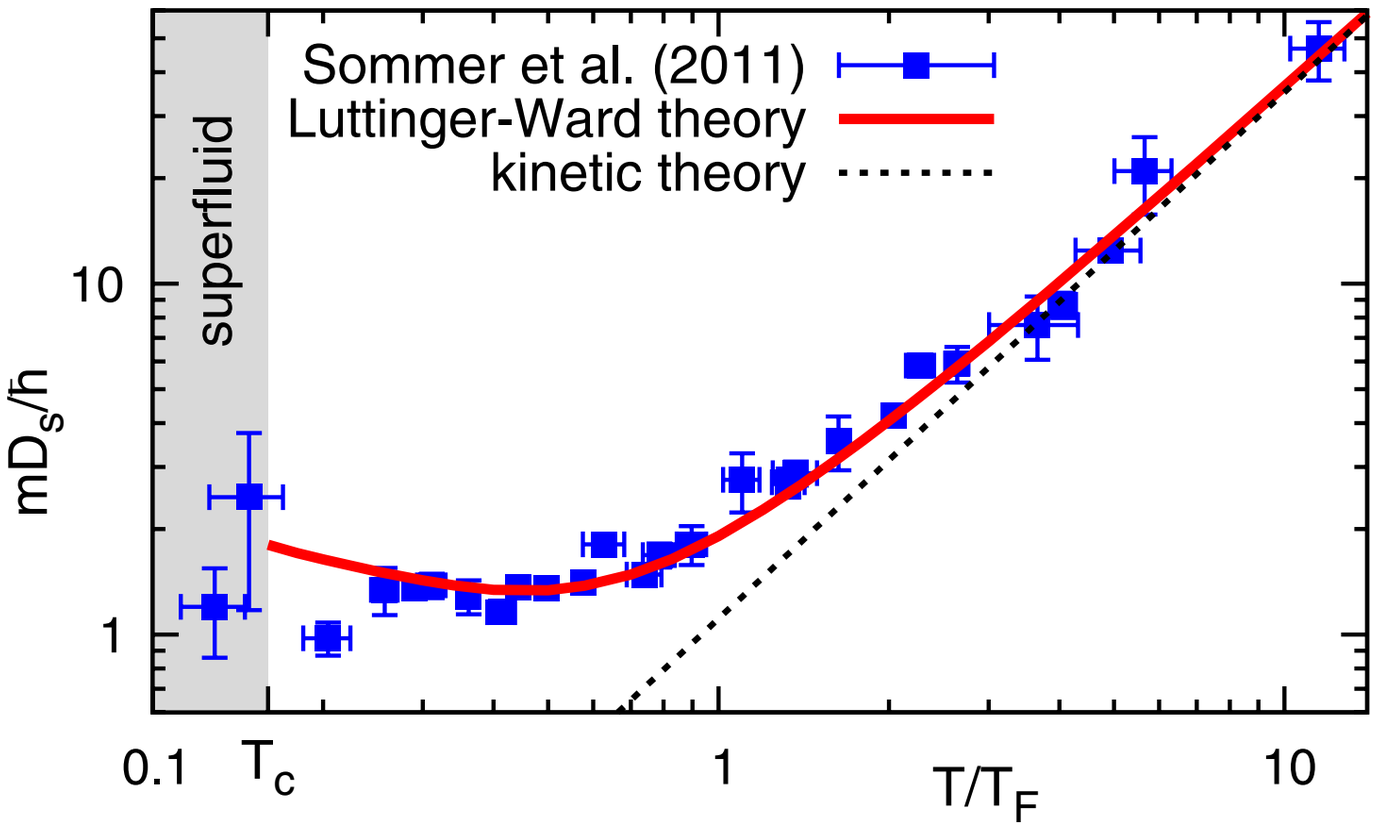
[Enss, Haussmann 2012]

- satisfies spin sum rule despite tail [Enss, arXiv:1209.3317]

$$\int \frac{d\omega}{\pi}\, \sigma_s(\omega) = \frac{n}{m}$$

# Spin diffusivity

- obtain diffusivity from conductivity, $D_s = \dfrac{\sigma_s(\omega = 0)}{\chi_s}$




(experiment rescaled from trap to infinite homogeneous box)

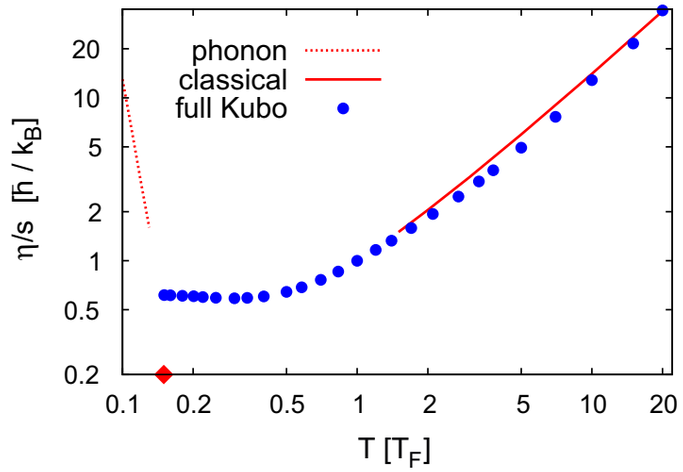
minimum $D_s \simeq 1.3\dfrac{\hbar}{m}$

[Enss, Haussmann 2012]

- recent Monte Carlo simulation: $D_s \gtrsim 0.8\dfrac{\hbar}{m}$ [Wlazlowski et al. arxiv:1212.1503]

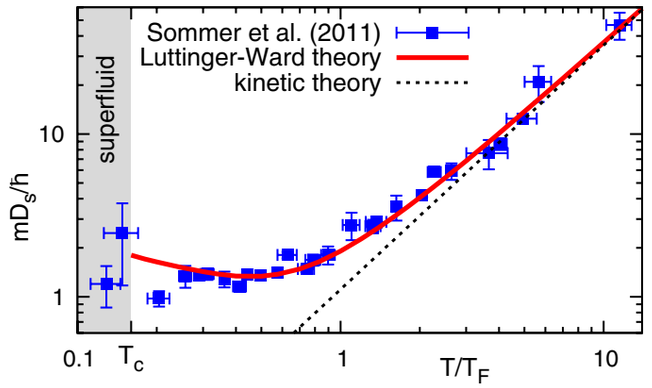
# Conclusion and outlook

- universal viscosity bound:
  unitary Fermi gas most perfect non-relativistic fluid




- clouds of opposite spin bounce off each other

- universal diffusion bound from holographic duality?
  unitary spin diffusivity $D_s \gtrsim 1.3\,\hbar/m$ constrains bound




- local transport measurements: [cf. Ku et al. 2012 for thermodynamics]
  extract diffusivity from spin-resolved dynamic structure factor