The Facets of Relativistic Quantum Field Theory[1]

H. G. Dosch[2] and V.F. Müller[3]

Abstract: Relativistic quantum field theory is generally recognized to form the adequate theoretical frame for subatomic physics, with the Standard Model of Particle Physics as a major achievement. We point out that quantum field theory in its present form is not a monolithic theory, but rather consists of distinct facets, which aim at a common ideal goal. We give a short overview of the strengths and limitations of these facets. We emphasize the theory-dependent relation between the quantum fields, and the basic objects in the empirical domain, the particles. Given the marked conceptual differences between the facets, we argue to view these, and therefore also the Standard Model, as symbolic constructions. We finally note that this view of physical theories originated in the 19th century and is related to the emergence of the classical field as an autonomous concept.

[1]published in **Eur. Phys. J H, 35 (2011), 331-376**
[2]Institut für Theoretische Physik der Universität, Philosophenweg 16, 69120 Heidelberg
[3]Fachbereich Physik der Technischen Universität,Postfach 3049, 67653 Kaiserslautern

Contents

0.1 Introduction

Relativistic quantum field theory is widely believed to provide a successful representation of subatomic physics as observed in the presently accessible experimental domain. In the course of time various approaches to quantum field theory have been developed, each focussing on different aspects of the goal envisaged. The development of these various approaches to quantum field theory reflects the quite indirect relationship between the basic concept in the theory and the central notion of the empirical domain, to wit, fields and particles. The experiments are bound to register *particles*, whereas the basic concept of the theory is the *quantum field*, technically seen an operator-valued distribution. Such a mathematical object requires in a quantum field theory showing interaction necessarily a process of renormalization, in contrast to a corresponding classical theory, where a field is a function. (In our paper we shall not discuss the general problems of quantum physics, as accentuated in the measuring process.) In general, the particle content of a quantum field theory is not related a priori to the field operators entering the theory, but is considered to result from the particular interaction of the quantum fields built into the theory. Nearly all of the observed subatomic particles are unstable, and there is a huge disparity in the respective lifetime of the different types of particles ranging from 10^3 to 10^{-25} seconds.

The object of this paper is to point out that the different approaches to quantum field theory alluded to above are rather different *facets* of a general program than distinct subtheories of a coherent embracing theory. To this end we shall first give a short overview of these different approaches to quantum field theory. The presentation chosen enters into the technical details only to such an extent, that it is still comprehensible for a physicist not working directly in the realm of elementary particle physics. This may lead sometimes to rather sketchy statements, but we shall quote always relevant original literature, specialized reviews and/or monographs. Already here it is important to stress that we do not intend a systematic presentation of the history of any development considered in this article, but we do mention time and again particular crucial steps in the progress of a development discussed. Likewise, our bibliography is necessarily selective, chosen in connection with our discussion.

In the overview we start with the more abstract approaches Axiomatic quantum field theory and Algebraic quantum field theory, now frequently named *General Theory of Quantum Fields* and *Local Quantum Physics*, respectively. These approaches intend to provide a solid basis of a quantum field theory on a rigorous mathematical footing, which is to serve as a framework for developing detailed theories related to phenomena. Hence, this foundational work mainly deals with general structures, exploiting in particular the basic locality property of a quantum field or of a quantum observable. Up to now one only knows quantum field theories without interaction, satisfying the basic assumptions of these approaches; the mathematically rigorous construction of a quantum field theory with interaction (in four dimensions) still remains an open problem. To settle this problem of primary theoretical importance is the ultimate goal of *Constructive quantum field theory*, dealing with it in a Euclidean formulation of quantum field theory. Briefly, the foundational efforts of these three approaches are mainly of paradigmatic character and largely ignore the relation to the concretely observed particles, apart from very general features. More closely related to particularities of the phenomena are *Renormalized Perturbation Theory* and *Lattice Regularization*, especially of a gauge theory, both approaches involving a concrete dynamical evolution. Each has its own specific inner concept of the relation between the quantized fields and the particles, which are more or less connected with observed particles.

As we look at quantum field theory as a physical theory, i.e. as a theoretical structure providing experimentally well corroborated results, it appears justified to confine our considerations to the development of relativistic quantum field theory, which eventually culminated in the Standard Model of particle physics. This model is considered to essentially cope with the physics of subatomic particles up to the currently accessible energies, which probe distances down to 10^{-16} cm. We therefore do not consider in our paper the large amount of recent research which is more speculative as e.g. noncommutative geometry of space-time, string theory, or other attempts to create a quantum theory of gravity[4].

The concept of an *internal symmetry* inherent in a concrete quantum field theory, which governs the interaction of various fields, has proven fertile, accounting for certain regularities observed in the phenomena. *Local gauge symmetry* is the most prominent instance of such a feature. The invariance of all interactions of the Standard Model under local gauge transformations of its quantum fields determines the form of this theory to a large extent. It should be noted that local gauge symmetry is not dealt with in Axiomatic

[4] For non-technical reviews see in (Seiler and Stamatescu (2007))

quantum field theory. Moreover, in Local Quantum Physics, which considers quantum observables as its basic concept, such a symmetry would remain hidden, since it leaves the observable quantities unchanged. To achieve a renormalized perturbation theory in the case of a field theory showing local gauge symmetry requires a specific extension of such a construction. The formulation of a field theory with local gauge symmetry within the approach of lattice regularization, fully keeping this symmetry, provides a *lattice gauge theory*. The mechanism of mass generation for gauge bosons by spontaneous symmetry breaking in the renormalized perturbation construction is in sharp contrast to lattice gauge theory, where spontaneous breaking of the gauge symmetry is not possible.

The Lagrangian of the Standard Model shows independent local gauge symmetries of its electroweak and its strong interaction part. Phenomenologigal consequences herefrom are extracted via the approaches of renormalized perturbation theory and of lattice regularization, according to the respective expected appropriateness of these approaches.

Effective Field Theories appear as particular perturbative constructions, possibly allowing non-renormalizability. A typical example of the latter case is Chiral Perturbation Theory.

Having presented a somewhat panoramic overview of the different approaches to quantum field theory we turn to an epistemological evaluation of the state of affairs we are faced with. We emphasize the marked differences in the respective set-up of these approaches, each showing an intrinsic concept of its own. Although restricted in scope, these approaches are directed towards a common ideal goal. Therefore we term them *facets* of quantum field theory and argue that they should be viewed as synchronous *symbolic constructions*. Moreover, we discuss implications of this view for the Standard Model. Should some day a mathematically consistent and empirically successful theory have been constructed, of which the facets are genuine parts, such an achievement should also be viewed as a symbolic construction, with a much larger domain of validity as any of the present facets, of course. We point out that the perspective of a symbolic construction emerged already in classical physics towards the end of the 19th century.

0.2 Particle Physics

0.2.1 Elementary Particles

From the experimental point of view, a microscopic particle manifests itself by a particular event in a special detection device, e.g. as a single track in a track chamber or as an electric discharge in a Geiger counter. A particle is identified mainly by its mass. The relation between energy E and momentum $\vec{p}$ leads to a discrete[5] value m, the mass of the particle:

$$E^2/c^2 - \vec{p}^{\,2} = m^2c^2, \tag{1}$$

where c is the velocity of light in the vacuum.

According to this definition, atoms and molecules are particles, and for many purposes, e.g. those of statistical mechanics, they can be considered even as elementary particles. In atomic physics, however, one has to take into account that atoms have an inner structure. The quantum mechanical version of the Rutherford atom, a nucleus surrounded by a cloud of electrons, explains an enormous range of phenomena from solid state physics to chemistry. The elementary particles in such a theory are the electrons, the constituents of the cloud, and the neutrons and protons, the constituents of the nucleus. This classification holds both from an experimental and from a theoretical point of view, since there is a well defined and unique relation between the theoretical objects, the vectors of a Hilbert space (states), and the observed particles. Composite particles, e.g. atoms and molecules, can be split into the elementary ones. The state of *composite* particles, e.g. an atom, is contained in the space constructed from elementary states as a superposition of products.

In spite of its still ongoing success, the nonrelativistic quantum mechanical model is unsatisfactory also from a purely phenomenological point of view for several reasons: 1) Nonrelativistic kinematics has only a limited range of validity and has to be superseded by relativistic kinematics. 2) In nonrelativistic quantum mechanics the number of elementary particles is conserved and a spontaneous decay cannot be described consistently; all states, also the

[5] Since charged particles cannot be separated from their radiation field, the energy of a charged particle is not discrete. Given the very weak coupling and the finite energy resolution of every counter, this problem is however experimentally not very relevant. Its serious theoretical implications and methods to overcome the difficulties are discussed at the end of sect. 0.4.2

excited ones, are stable in this theory. 3) Last but not least, it is necessary to describe also electromagnetism, which is inherently relativistic, in quantum physics. This led to the development of relativistic quantum field theory, the principles of which will be introduced in the next subsection.

Extending the range of energies in scattering experiments transmutation of energy into matter and *vice versa* appeared as new salient feature in the reaction processes. Particles can be created and annihilated, this process is governed by certain conservation laws, in particular the energy equivalent $\Delta E = m c^2$. In the course of time more and more particles, which could be called elementary particles, were detected[6]. All of them are unstable, but some of them have a lifetime which is so long that the uncertainty in energy is unmeasurably small (for charged pi-mesons the relative mass spread $\Delta m / m \approx 1.8 \cdot 10^{-18}$). But there are other ones, which have an unmeasurably short lifetime, but where in turn the spread in energy in eq. (1) is clearly evident[7]. There are therefore two possibilities to define an elementary particle in purely experimental terms:

1 An entity which is identified by a particular event and with a mass spread which is small compared to the mass, determined according to (1). These particles can be used for constructing beams in scattering experiments and show, if charged, a visible track in a track chamber (Wilson, bubble, or wire chamber).

2 An object with a distinct bump in the mass distribution.

In the latter case the particle can only be detected indirectly, e.g. by its decay products. The second definition includes definition **1**. These particles are listed in the "Particle Listing" in the standard bi-annual "Review of Particle Physics",the last issue is (Nakurama, K. *et al.* (2010)).

The definitions, especially **2**, are rather vague. Among other things this is reflected in the disappearance and reappearance of particles in the "Particle Listing"(Nakurama, K. *et al.* (2010)). The definitions will be supplemented below by other ones based mainly on theoretical arguments.

It seemed first tempting to identify the states with broader energy distribution as composite ones, like excited atoms. This goal could certainly not

[6]The first ones were the muon (Anderson (1933)) and the charged pi-meson(Lattes *et al.* (1947))

[7]The first one was the Δ-resonance in pion-nucleon scattering(Anderson *et al.* (1952)). Its mass is 1.232 GeV/c^2, its width $\Delta m = 120$ GeV/c^2

be achieved within nonrelativistic quantum mechanics and has been another motivation for the search of a relativistic quantum theory.

0.2.2 Relativistic Quantum Field Theory

Relativistic quantum field theory has grown out by a synthesis of classical relativistic field theory, notably Maxwell's theory of electromagnetism, and the quantization rules used in nonrelativistic quantum mechanics. Like the latter it enables to make quantitative statements and predictions about the outcome of experiments, that is of probabilities to observe particles under well defined experimental conditions.

As in the classical relativistic theory, space-time is fixed as a four-dimensional real vector space, with a metric invariant under Lorentz transformations and translations, that is the elements of the Poincaré group. In contrast to nonrelativistic quantum mechanics, however, relativistic quantum field theory implies the possibility to create and annihilate particles.

The basic entities of the theory, the quantum fields, are space-time dependent operators in a Hilbert space. There is however no point-like mapping of the space-time continuum to the field operators, but only as a distribution. Phrased technically, a relativistic quantum field is an operator-valued distribution. Therefore a well-defined operator cannot be related to a definite space-time point x, but only to a space-time domain of finite, though arbitrarily small, extent.

In order to make use of the quantization rules and to incorporate the causal space-time structure together with the Poincaré symmetry into the theory, one starts with a Lagrangian formulation of the classical field theory. The incorporation is achieved by keeping the functional form of the classical Lagrangian density — a *local* polynomial of the field(s) and of its space-time derivatives — and by just replacing the classical field $\varphi(x)$ by the quantum field $\phi(x)$.

As an example for immediate reference we show the Lagrangian density of a model involving a single real scalar field only:

$$\mathcal{L}(x) \;=\; \frac{1}{2}\left(\partial_\mu\phi(x)\,\partial^\mu\phi(x) - m^2\phi^2(x)\right) - \frac{\lambda}{4!}\phi^4(x), \tag{2}$$

where the first part describes a free field and the term proportional to the coupling constant λ the (self)interaction of the field.

Due to the fact that the quantum fields behave like distributions, not like functions, the local products of field operators are ill-defined and lead to infinities, if one attempts to deduce directly physical consequences from such a Lagrangian. In the early period of exploring quantum field theories these infinities caused a considerable bewilderment. Later, a systematic reformulation of the originally ill-defined approach to relativistic quantum field theory was achieved — called perturbative renormalisation theory — which provides a mathematically well-defined quantum field theory as a formal power series in a renormalised version of the coupling constant. This theory produces strictly finite physical predictions in each order of the expansion. The price to be paid will be expounded in the sequel.

0.2.3 Particles and Fields

Originally the fields introduced in quantum field theory were thought to be operators in a state space close to that of quantum mechanics, constructed to describe the states of elementary particles. It turned out, however, that this procedure is to restrictive.

A free field theory is a theory without interaction between the fields, $\lambda = 0$ in our example (2). In this case we can establish a direct relation between particles and fields. The state space is constructed on the basis of single particle states similar to quantum mechanics. In quantum mechanics operators of observables act in an n-particle space, which is the properly symmetrized or antisymmetrized product of single particle spaces, whereas quantum fields act in the *Fock space*, the direct sum of all spaces with a discrete number of particles, including the vacuum, the zero-particle state. Free field theory is the basis of renormalized perturbation theory (sect. 0.3.4) and therefore the relation between particle and field in perturbation theory is close to that of the free theory. Seemingly, regarding the Lagrangian (2), we can therefore use another definition:

3 A physical particle corresponds to a field in the quadratic part of the Lagrangian.

It should be noted, however, that important caveats are necessary. In the important case of a gauge field theory not all components of the field are independent dynamical variables; the consequences will be discussed in more detail in section 0.4.2.

Since perturbation theory can be fruitfully applied in the theory of electromagnetic and weak interactions, we can give a closely related definition, which is nearer to experiment, namely

4 a particle is an entity which were stable, if there was no weak and electromagnetic interaction

This definition is very close to the practical definition **1** of sect. 0.2, which can be formulated in a more precise mathematical way as

1' a particle corresponds to a particular unitary representation of the Poincaré group (Lorentz transformations and translations), implying definite mass and spin.

0.3 The Facets of Quantum Field Theory

In the course of time various approaches to relativistic quantum field theory have been developed, each focussing on different aspects of the goal envisaged. The *General Theory of Quantum Fields* and *Local Quantum Physics* deal with the basic conceptual frame and general consequences therefrom. *Constructive Quantum Field Theory*, *Perturbative Renormalization Theory*, and *Lattice Theory* direct their attention to the construction and evaluation of concrete dynamical models. We call these different approaches *facets* and give first a short overview.

0.3.1 General Theory of Quantum Fields

In the middle of the last century, the achievement of quantum electrodynamics (QED) as a renormalized perturbation theory together with the insight gained into inherent difficulties of this development prompted a new approach, often termed *Axiomatic Quantum Field Theory.* The aim of this approach is to formulate a mathematically well-defined general conceptual frame of a relativistic quantum field theory beyond perturbation theory, and to analyze its consequences, (Streater and Wightman (1980); Jost (1965)). This frame is formed by a few postulates - usually called *Wightman axioms* - avoiding any recourse to classical field theory. These postulates essentially are
W1.(Relativistic Covariance) The Hilbert space $\mathcal{H}$ of physical states carries

a unitary representation $U(\Lambda, a)$ of the restricted Poincaré group, i.e. with Λ a restricted Lorentz transformation[8] (without space and time reflections) and a a space-time translation. There is a unique vacuum state Ω which satisfies

$$U(\Lambda, a)\,\Omega = \Omega\,, \quad \text{for all } \{\Lambda, \mathrm{a}\}\,. \tag{3}$$

W2.(Energy-Momentum Spectrum) The generators $(H = P^0, \vec{P})$ of the translation subgroup $U(1, a) = \exp iP^\mu a_\mu$ have spectral values $(p^0 > 0, \vec{p})$,

$$(p^0)^2 - \vec{p}^{\,2} \geq 0\,. \tag{4}$$

W3.(Field Operator) The quantum field $\Phi_j(x)$, when smeared with a smooth test function $f(x)$ on space-time having *compact* support, formally,

$$\Phi_j(f) := \int d^4x\, f(x)\, \Phi_j(x)\,, \tag{5}$$

is a proper operator in $\mathcal{H}$ on a dense domain containing the vacuum state Ω. This vector Ω is *cyclic* with respect to $\mathcal{P}$, the set of all polynomials of smeared fields. That is, $\mathcal{P}\Omega$ is dense in $\mathcal{H}$. A precise formulation of (5) is to regard the field $\Phi_j(x)$ as an operator-valued distribution: for all vectors Ψ, Ψ' the scalar product in $\mathcal{H}\,, (\Psi, \Phi_j(f)\Psi')$, is a functional of f.
W4.(Covariant Field Transformation) The (in general) multi-component field transforms covariantly under restricted Poincaré transformations,

$$U(\Lambda, a)\,\Phi_j(x)\,U(\Lambda, a)^{-1} = \sum_{k=1}^{n} S_{jk}(\Lambda^{-1})\Phi_k(\Lambda x + a)\,, \tag{6}$$

where S is a finite-dimensional represention of the restricted Lorentz group, see footnote in W1.
W5.(Locality) If the supports of $f(x)$ and $g(y)$ are space-like separated, the fields either commute $(-)$, or anticommute $(+)$,

$$\Phi_j(f)\Phi_k(g) \mp \Phi_k(g)\Phi_j(f) = 0\,. \tag{7}$$

The vacuum expectation values, for simplicity written in the case of a scalar field, $n \in \mathbf{N}$,

$$(\Omega, \Phi(x_1)\Phi(x_2), \cdots, \Phi(x_n)\,\Omega) =: W_n(x_1 - x_2, x_2 - x_3, \cdots, x_{n-1} - x_n)\,, \tag{8}$$

[8] More precisely, in order to account also for half-integer spin the restricted Lorentz group $L_+^\uparrow$ has to be replaced by its universal covering group $SL(2, C)$.

play a central role in exploring the framework: It has been shown (Streater and Wightman (1980)) that from the collection of these distributions (8) - called *Wightman functions* - the theory satisfying the postulates W1-W5 can be reconstructed.

From these postulates several physically important structural results have been deduced (Streater and Wightman (1980)). We mention the *PCT*-theorem which asserts that the combined action of time reflection T, particle-antiparticle conjugation C and space reflection P is necessarily a symmetry, whether the individual actions are symmetries or not. Furthermore, a distinct connection between spin and statistics results: the alternative in W5, (7), is reduced requiring there an integer spin field to commute, but a half-integer spin field to anti-commute. Moreover, it has been shown that within this frame a collision theory of particles can be formulated for massive aymptotic states. However, there is no direct correspondence between the field operator and the particle content of the theory. The consequences of the Wightman postulates are mainly derived by analyzing analytic properties of the Wightman functions (8). These “functions”, actually distributions with respect to the real coordinate differences $x_l - x_{l+1}$, due to W1-W5 can be analytically continued to complex values with a large domain of holomorphy. This domain contains the so-called Euclidean points $z_l = (is_l, \vec{x}_l)$ with $s_l \in \mathbf{R}$, $\vec{x}_l \in \mathbf{R}^3$. Moreover, the Wightman functions are determined by their values at these points which allows to define the *Schwinger functions* at non-coinciding points,

$$S_n(x_1^E - x_2^E, \cdots, x_{n-1}^E - x_n^E) = W_n(z_1 - z_2, \cdots, z_{n-1} - z_n) \tag{9}$$

with $x_l^E = (s_l, \vec{x}_l)$ and $x_k^E \neq x_l^E$, $k \neq l$. The set of Schwinger functions constitutes the *Euclidean formulation* of the original relativistic quantum field theory, with the Euclidean group taking the place of the Poincaré group. In the reverse direction, Osterwalder and Schrader (1973); 1975) have established *Euclidean axioms* characterizing autonomously given Euclidean correlation functions, i.e. supposed Schwinger functions, to determine a relativistic quantum field theory. The Euclidean formulation allows to employ powerful constructive techniques as e.g. functional integration, see sect. 0.3.3.

Clearly, a *free* quantum field theory, i.e. a theory without interaction, satisfies the Wightman postulates. However, a mathematically rigorous construction (i.e. beyond formal perturbation theory) of a concrete interacting model in four space-time dimensions has not yet been achieved. Nevertheless, it is widely believed, that such a construction would have to comply with the Wightman postulates. A qualification has to be made, however. To encompass a *gauge field theory* the Wightman postulates partly have to be modified,(Strocchi (1993)).

0.3.2 Local Quantum Physics

The term *Local Quantum Physics* has taken the place of *Algebraic Quantum Field Theory* used before. This development presents an alternative conceptual frame to serve as a foundational basis of quantum field theory. Whereas the general formulation of Axiomatic quantum field theory is based on the notion of a relativistic quantum field, the basic notation of Local Quantum Physics, see (Buchholz and Haag (2000)), is directly related to the ultimate goal of a physical theory, namely to account for observations. Local Quantum Physics therefore introduces the notion of a *local observable* as its fundamental theoretical construct: it is localized in space-time and can be subject to relativistic causality. Stated more technically, the basic concept is a system of local observables which are elements of a C^*-algebra:[9] To every bounded open region $\mathcal{O}$ of space-time is attributed a C^*-algebra $\mathcal{A}(\mathcal{O})$, describing the physical *observables* of this region. These algebras are assumed to show the following three properties:

i) The algebra of observables on a space-time region $\mathcal{O}_2$, which contains the region $\mathcal{O}_1$ contains the algebra on the smaller region: if $\mathcal{O}_1 \subseteq \mathcal{O}_2$, then $\mathcal{A}(\mathcal{O}_1) \subseteq \mathcal{A}(\mathcal{O}_2)$.

ii) The formulation is relativistic invariant, which appears as a representation of the Poincaré transformations by automorphisms $\sigma_{(a,\Lambda)}$ of the algebra. Denoting by $\mathcal{O}_{(a,\Lambda)}$ the transformed region $\mathcal{O}$, then $\sigma_{(a,\Lambda)}(\mathcal{A}(\mathcal{O})) = \mathcal{A}(\mathcal{O}_{(a,\Lambda)})$.

iii) If the space-time regions $\mathcal{O}_1$ and $\mathcal{O}_2$ are space-like separated, then each element of $\mathcal{A}(\mathcal{O}_1)$ commutes with each element of $\mathcal{A}(\mathcal{O}_2)$. (This property is sometimes called "Einstein causality".)

To obtain physical predictions quantum mechanical expectation values have to be generated from the (abstract) algebra $\mathcal{A}$ of observables. This is achieved by a normalised positive linear functional ω on $\mathcal{A}$, called a physical *state* of the system considered. Hence this state maps the elements of the

[9] In quantum mechanics observables are formed by self-adjoint operators, in general unbounded ones. Restricting their spectral representation to finite spectral intervals leads to bounded operators. The set of all bounded (linear) operators of a complex Hilbert space forms a C^* algebra.

algebra into complex numbers with

$$\omega(\mathbf{1}) = 1, \quad \omega(A^*A) \geq 0\,, \quad \omega(\alpha A + \beta B) \,=\, \alpha\,\omega(A) + \beta\,\omega(B) \in \mathbf{C}\,, \qquad (10)$$

where $\alpha, \beta \in \mathbf{C}$ and $\mathbf{1}, A, B \in \mathcal{A}$. In its physical interpretation $\omega(A)$ is the expectation value of the observable A in the state ω. Given an algebra of observables $\mathcal{A}$ and a state ω, then a well-defined mathematical construction, the Gel'fand-Naimark-Segal (GNS) representation, determines a concrete representation of this algebra $\mathcal{A}$ by linear bounded operators on a Hilbert space.

In this approach quantum fields have no basic status, as they do not directly represent physically observable quantities. They may and can act as particular building blocks in forming observables of the algebra $\mathcal{A}$.

Among the principal achievements of local quantum physics (Kastler (1990); Haag (1996)) is a general characterisation of a relativistic quantum field theory at finite temperature (the Kubo-Martin-Schwinger (KMS) condition). Another important achievement is the Doplicher-Haag-Roberts (DHR) theory of superselection sectors. This theory provides an analysis of the charge structure completely within the algebra of observables, i.e. it treats charged sectors without introducing charged field operators to create these sectors. Seen technically, the superselection sectors are inequivalent representations of the algebra of observables. The analysis has been extended to cover an internal *global* symmetry group, but not a *local* one. Local that is gauge symmetries (see sect. 0.4.2) seem to be of prime importance in a physically realistic theory as will be discussed in sect. 0.5. By definition, however, observables are invariant under these transformations. Therefore, in an approach strictly based on the concept of observables the question arises, which intrinsic properties reveal, that a theory is actually a gauge theory?

Besides this foundational work rooted in the concept of observables in recent times the perturbative construction of a quantum field theory in a classical curved space-time, as given by General Relativity, in place of Minkowski space has been investigated within the algebraic framework,(Brunetti and Fredenhagen (2009); Wald (2009)). A generic space-time manifold $\mathcal{M}$ has no symmetries and its metric g depends on time. Then, because of lack of symmetry under time translations there is no longer a natural notion of a vacuum state and of a particle, which originates from positive-frequency solutions of a (free) wave equation. Considering the family of globally hyperbolic time-oriented manifolds[10] the fundamental locality property iii) can be kept. The (formal) perturbative construction of an interacting quantum field theory is based on a free one, the latter is determined by its two-point function. As a substitute for the two-point function in Minkowski space, which results from the vacuum state, a two-point function $H(x_1, x_2)$ of *Hadamard type* is constructed on $\mathcal{M}$. Such a singular function, i.e. a distribution, has a similar singularity structure in the coincidence limit $x_1 \to x_2$ as its equivalent in Minkowski space, but is not unique. Two Hadamard functions H and H' have a smooth difference $H - H'$, leading to equivalent theories. In the case of a self-interacting scalar field a renormalized (formal) perturbation construction has been achieved (Brunetti and Fredenhagen (2000); Hollands and Wald (2002)). Poincaré invariance in Minkowski space restricts the singular terms arising and to be dealt with by the process of renormalisation. This role is taken by a crucial principle of general covariance, which requires a purely local and covariant construction out of the space-time geometry. In consequence, the theory is constructed on the whole family of space-time manifolds considered. Up to now, this theory is interpreted in terms of covariant fields. Physical predictions would in addition require to point out (in principle) measurements aimed at by the theory as well as to define the corresponding state functionals ω.

0.3.3 Constructive Quantum Field Theory

In the late sixties, faced with the demands of the Wightman postulates and the mathematically unsatisfactory status of renormalized perturbation theory based on a formal series expansion, likely not to be summable, a new branch of theoretical activity set in, soon called Constructive Quantum Field Theory. Its general aim is to construct with mathematical rigour concrete models of quantum field theory resulting from specific Lagrangians with interaction. Up to now this aim has been approached to a notable, but limited extent. Within the Euclidean formulation, elaborate techniques of functional integration - the mathematical version of Feynman's path integral - have been developed providing the rigorous construction of a number of models, among them with the Lagrangian (2), (Glimm and Jaffe (1987)), but in two and three space-time dimensions only. Via the Osterwalder-Schrader reconstruction these models satisfy the Wightman axioms with corresponding lower-dimensional

[10]It includes the Robertson-Walker metric, which is of prime cosmological interest.

space-time. In the language of perturbation theory the models constructed are super-renormalizable, i.e. the inherent singular ultraviolet behaviour only appears in terms of low order in the formal series expansion (see next subsection)). Nevertheless, the crucial renormalization outside perturbation theory had to be achieved in the rigorous construction of these models.

Viewed from perturbation theory, the massive Gross-Neveu model in two space-time dimensions differs characteristically from the models referred to before: it is renormalizable, but not super-renormalizable, and ultraviolet asymptotically free (see next subsection). These properties are also shown by a non-Abelian gauge theory in four space-time dimensions, attracting additional physical interest in the Gross-Neveu model. It is built of a multiplet of Dirac fields $\psi^a(x)$ with an inner symmetry index $a = 1, 2, \cdots, N \geq 2$ and has the classical Lagrangian density

$$\mathcal{L}(x) = \sum_{a=1}^{N} \overline{\psi^a}\,(i\gamma^\mu \partial_\mu - m)\,\psi^a + \frac{\lambda}{N}\Big(\sum_{a=1}^{N} \overline{\psi^a}\,\psi^a\Big)^2 , \qquad (11)$$

with a quartic self-interaction. Proceeding in the Euclidean formulation the Schwinger functions of this model have been rigorously constructed,(Gawedzki and Kupiainen (1985); Feldman *et al.* (1986); Disertori and Rivasseau (2000)), satisfying the Osterwalder-Schrader axioms (for fermions). Furthermore, these Schwinger functions are shown to be the Borel sum of their renormalized perturbation series. This is interesting for renormalized perturbation theory (sect. 0.3.4), since it shows that in this case the *formal* power series can be brought into an explicit form by resummation.

0.3.4 Perturbative Renormalization Theory

This approach uses the well-defined theory of "free" quantum fields as its basic frame and deals with the interaction as a "small" perturbation. It was mainly focussed on quantum electrodynamics at the beginning. In the perturbation expansions as actually performed soon infinities occured, which were considered as a serious obstacle. Eventually, these infinities could be "cancelled" by an approriate redefinition of the parameters entering the theory (mass, coupling constant, field normalization), a procedure called *renormalization*. This development is extensively treated by Schweber (1994). After the advent of a covariant perturbation formalism (Feynman, Schwinger, Tomonaga), the era of systematic perturbative renormalization opened with

Dyson's pioneering work (Dyson (1949a); 1949b)). In the course of two decades the rigorous Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ) version of perturbative renormalization theory has been achieved, (Bogoliubov and Parasiuk (1957); Hepp (1969); Zimmermann (1970)). Moreover, this version has been extended by Lowenstein (1976), to BPHZL, covering now massless fields, too. There are other formulations, physically equivalent, but differing technically (e.g. using counterterms or dimensional regularization). Common to the BPHZL formulation and the other ones is to exploit the translation symmetry of the Minkowsky space and passing via Fourier transformation to momentum space. In contrast, Epstein and Glaser (1973), have developed an inductive construction of the S-operator in terms of the free field operator using crucially its locality property. The role of renormalization is replaced there by an operation of distribution splitting. As the construction does not require translation invariance to hold, it can also be performed, if an external (classical) field is present, (Dosch and Müller (1975)), or for investigating quantum fields on a given curved space-time, (Brunetti and Fredenhagen (2000)). The perturbation expansion of a gauge field theory (sect. 0.4.2) demands in addition to cope with the inherent local gauge invariance, requiring a particular construction. Eventually, t'Hooft and Veltman, ('t Hooft (1971b); 1971a); 't Hooft and Veltman (1972a)), demonstrated the perturbative renormalization of a spontaneously broken non-Abelian gauge theory, an achievement of prime physical importance in view of the Standard Model.

We outline the salient points of perturbative renormalization theory in the case of a neutral scalar field, with classical Lagrangian (2). The basic quantities to be determined by perturbation theory are the *time-ordered* Green functions,

$$\tau_n(x_1, x_2, \cdots, x_n) := (\Omega, \mathcal{T}\Phi(x_1)\Phi(x_2), \cdots, \Phi(x_n)\,\Omega)\,, \tag{12}$$

the vacuum expectation values of time-ordered products of field operators,

$$\begin{aligned} \mathcal{T}\Phi(x_1)\Phi(x_2), \cdots, \Phi(x_n) \;&=\; \Phi(x_{j_1})\Phi(x_{j_2}), \cdots, \Phi(x_{j_n})\,, \\ &\qquad x^0_{j_1} > x^0_{j_2} > \cdots > x^0_{j_n}\,. \end{aligned} \tag{13}$$

In spite of the distinction of the time component x^0 of a space-time point x, due to the locality property of the field operator $\Phi(x)$ this product is relativistically covariant. However, at coinciding points the definition has to be supplemented by way of construction.[11] Moreover, it suffices to consider connected Green functions, i.e. those which do not factorize. Because of translation invariance, the Fourier transformed connected Green functions (12) have the form

$$\Big(\prod_{j=1}^{n}\int dx_j e^{iq_jx_j}\Big)\,\tau_n(x_1,x_2,\cdots,x_n)^{con} = (2\pi)^4\,\delta(q_1+\cdots+q_n)\,G_n(q_1,\cdots,q_n)\,, \tag{14}$$

the n four-momentum variables of G_n satisfying $q_1+\cdots+q_n=0$.

In perturbation theory the Green functions are generated as formal power series in the coupling

$$G_n(q_1,\cdots,q_n) = \sum_{r=1}^{\infty}\sum_{F}\,\lambda^r I^{(r)}_{n,F}(q_1,\cdots,q_n) \tag{15}$$

The terms $I^{(r)}_{n,F}(q_1,\cdots,q_n)$ emerge from the local interaction and from the *propagators*, the two point functions of the free theory. These expressions are multiple momentum-space integrals, with integrands formed of products of couplings and of propagators. They can be represented graphically by *Feynman diagrams.* Perturbative renormalizability is determined by the local form of the Lagrangian and the resulting behaviour of these integrands for large momenta.

The inductive generation of the Green functions proceeds in two steps: an *intermediate regularization* followed by a process of *renormalization.*

(i) The regularization has to prevent divergences to occur: in d-dimensional space-time, an unregularized Feynman integral $I^{(r)}_{n,F}$ contributing to G_n in the perturbative order r has the *superficial degree of divergence*, in the case of Lagrangian (2),

$$\omega(I^{(r)}_{n,F}) = d-(\tfrac{1}{2}\,d-1)+(d-4)\,r\,. \tag{16}$$

For the case $\omega<0$ the integral appears to converge, for $\omega=0$ to diverge logarithmically or to converge, and for $\omega>0$ to diverge. The qualification *superficial* means that a Feynman integral with $\omega<0$ may also diverge, if it contains a divergent subintegral. Regarding (16) we notice, that for

[11] Formally, a time-ordered Green function is the product of the corresponding Wightman function with step functions of time differences, requiring additional properties to be defined.

$d = 2, 3$ a non-negative ω can appear only in low order r: the respective theory is *super-renormalizable.* In the physical case $d = 4$ a degree $\omega \geq 0$ can arise in any order r of the perturbation expansion, but only if $n = 2$ or $n = 4$: the theory is *renormalizable.* In contrast, if $d > 4$ for every n the degree eventually becomes positive with increasing order r : the theory is *non-renormalizable.* A variety of equivalent regularization methods have been invented. As regards Yang-Mills theories, however, only dimensional regularization appears practicable since it respects the local gauge symmetry. Dimensional regularization consists in the analytic continuation of the space-time dimension d in Feynman integrals, and the fact that these integrals converge for sufficiently small d, see (16).

(ii) Renormalization is an inductive procedure to subtract from the regularized Feynman integrals their potentially divergent parts such that the difference stays finite upon removing the regularization. In a renormalizable theory as key property these subtractions can be generated by introducing additional local interaction terms into the Lagrangian, ("counter terms") which exactly correspond to all the local terms composing the (classical) Lagrangian (2), but supplied with "coupling constants" which themselves are formal power series in the expansion parameter λ, with coefficients depending on the regularization,(Zimmermann (1969)). In every order the subtraction operation is only determined up to a finite number of constants, (three in the case considered here, see (2)), which have to be fixed by prescribing related *renormalization conditions.* A massive theory allows to subtract on mass-shell, yielding physical parameters characterizing the theory.

Scattering processes are expressed by the S-matrix of the theory. The S-matrixelement describing a process of 2 incomming particles with momenta p_1, p_2 and $n - 2$ outgoing particles with momenta $p_3, \cdots, p_n$, where $p = (\sqrt{m^2 + \vec{p}^2}, \vec{p})$, is given by the Lehmann-Symanzik-Zimmermann (LSZ) reduction formula, (Lehmann *et al.* (1955)), in terms of the time-ordered Green function (14),

$$(p_3, \cdots, p_n|S - 1|p_1, p_2) = \qquad (17)$$
$$\delta(p_1 + p_2 - p_3, \cdots - p_n) \lim \prod_{j=1}^{n} \frac{(p_j^2 - m^2)}{\sqrt{Z}} G_n(p_1, p_2, -p_3, \cdots, -p_n) \, ,$$

where lim denotes the mass shell limit $p_j^0 \to \sqrt{m^2 + \vec{p_j}^{\,2}}$. The space of physical states is the Fock space connected with a free neutral scalar field operator, the field strength renormalization Z follows from the renormalization conditions.

In the case of a massless theory, in particular in an unbroken non-Abelian gauge theory, the subtraction in the renormalization procedure cannot be performed on mass-shell, because of infrared singularities. The classical Lagrangian lacks any scale parameter, cp. (2) with $m = 0$ (in $d = 4$). In the metamorphosis to a quantum field theory, however, such a parameter necessarily has to appear in the renormalization conditions. The renormalization point is appropriately chosen at Euclidean values of the momenta, $p^2 = -\mu^2 < 0$. Independence of observable quantities from the arbitrary choice of μ leads to constraints, expressed by the renormalization group. This holds in particular for the effective coupling strength $\lambda(\mu)$. A non-Abelian gauge theory is distinguished by the property, called *asymptotic freedom*, that its effective coupling strength vanishes with $\mu \to \infty$.

In comparision with other approaches to quantum field theory perturbative renormalization theory appears most fertile in providing physical predictions, both in quantity and in precision. It was mainly the very high degree of agreement between the outcome of specific experiments and its description by QED, which led to the recognition of the perturbative method, in spite of serious unmet questions: Mathematically, the construction produces inductively a formal power series (only) without attention to convergence. In practice only few orders are explicitly calculable.

0.3.5 Lattice Regularization

Field theories on a lattice were introduced in order to obtain vacuum expectation values of interacting field theories in a constructive way by quadratures. For that one discretizes the space-time continuum to a lattice, the fields are attached to the lattice points (and possibly to the links between the points). The n-point functions, the vacuum expectations values of quantum field theory, are obtained by integration over the fields, with the measure given by the action. In order to obtain a well defined, i.e. positive, measure, one has to construct a Euclidean theory: the lattice is not embedded into a four-dimensional space-time continuum with Minkowski metric, but into an Euclidean four-dimensional space. In this way one obtains not the Wightman, but the Schwinger functions, see (9). Schematically we can write the expression for the Schwinger n-point function as:

$$\langle\phi[k_1]\dots\phi[k_n]\rangle = \frac{1}{Z}\int\prod_i d\phi[i](\phi[k_1]\dots\phi[k_n])e^{-S_E} \tag{18}$$

Here $\phi[j]$ is the field at the lattice point x_j, Z is the normalization for the measure and S_E is the action obtained from the discretized classical Euclidean Lagrangian; for a hypercubic lattice with lattice spacing a one has:

$$S_E = a^4\sum_i \mathcal{L}[\phi[i]], \tag{19}$$

the index i runs over all lattice points. This shows the close connection between the lattice version of relativistic quantum field theory and classical spin models of statistical mechanics.

If the lattice is finite one has introduced an infrared (long distance) cutoff, and an ultraviolet (short distance) cutoff because of the finite lattice spacing. In this way it is possible always to work with well defined quantities. One obtains hopefully a continuum theory by making the lattice infinite (thermodynamic limit) and letting the lattice spacing a tend to zero (continuum limit).

Up to now lattice theories are the only way to explore constructively strongly interacting theories. A major task is to deduce the observed properties of compound particles from the theory. Massive particles correspond to correlation lengths in the Schwinger functions. These correlation lengths are, in contrast to the lattice spacing, physically sensible quantities and set the scale.

A major task is to come to a physically sensible continuum limit. On the lattice the correlation length ξ is expressed in units of the lattice spacing, that is the continuum limit $a \to 0$ corresponds to $\xi \to \infty$ on the lattice. In the language of statistical mechanics the continuum limit corresponds to a critical point of the (Euclidean) theory. We shall come back to lattice theories in the form of lattice gauge theories in sect. 0.4.3.

0.4 Internal Symmetries

0.4.1 General Remarks

The insufficiency of perturbation theory in the case of strong interactions and certain phenomenological regularities led to the concept of (approximate) *internal* symmetries. In contrast to the Poincaré symmetry (see sect. 0.3.1) these symmetries are not related to the space-time structure. An (exact) internal symmetry of a field theory is characterized by a continuous symmetry group (Lie group) $\mathcal{G}$: n fields are joint to constitute a multiplet, i.e. to each member is attributed a basis vector of an irreducible unitary representation of the symmetry group. The representation is a map of the group $\mathcal{G}$ to n-dimensional unitary matrices ω:

$$\mathcal{G} \ni h \to \omega \in \mathrm{Mat}(C, n) \tag{20}$$

The n-dimensional space is called *charge space* and the fields *matter fields*. The theory is said to be symmetric under the group $\mathcal{G}$ if the Lagrangian is invariant under the transformations ω of the matter fields. If there are terms in the Lagrangian which are not invariant under these transformations, the symmetry is said to be broken by these terms.

The historically first and still very important example of such a symmetry is the isospin symmetry, for historical remarks see Kemmer (1982). The (exact) isospin symmetry is an idealization and broken at least through the electromagnetic interaction. The symmetry breaking is much stronger for another internal symmetry, the so called flavour $SU(3)$.

If the classical Lagrangian is invariant under $\mathcal{G}$, but if higher order perturbation corrections break the invariance, one say the symmetry is broken by an *anomaly*.

A symmetry is called *spontaneously broken*, if the Lagrangian is invariant under symmetry transformations, but not the vacuum (cf sect. 0.3.1), the ground state of the theory. The most intuitive example is the ferromagnet. In the absence of an external field the Lagrangian is invariant under rotations, but not the ground state, the magnetic field of which points in a distinct direction.

An important consequence of the spontaneous breaking of a continuous symmetry is the occurrence of massless bosons, the so called Goldstone bosons (Goldstone (1961); Goldstone *et al.* (1962)). They are present if a continuous symmetry of the Lagrangian with group $\mathcal{G}$ is spontaneously broken to the (smaller) symmetry of the vacuum state, with group $\mathcal{H} \subset \mathcal{G}$, and if there are non-vanishing vacuum expectation values of operators with non-trivial quantum numbers. The fields corresponding to the Goldstone bosons are the coordinates in the coset space $\mathcal{G}/\mathcal{H}$. In the case of the ferromagnet the full 3-dimensional rotational symmetry is broken to a rotation symmetry around the direction of the magnetic field of the ferromagnet. The two resulting Goldstone bosons are the quanta of the transverse spin-wave field.

A general formulation of the theorem for relativistic local fields is given in (Strocchi (2008)), ch. II.17. It states: "If a continuous symmetry, which is represented by a one parameter group of *-automorphisms of the field algebra locally generated by charges is spontaneously broken in the sense, that for at least one field operator A the vacuum expectation value is different from zero, then there exists in the Fourier transform of the vacuum expectation value of the product of that field A and the charge density a $\delta(p^2)$ singularity." This singularity signifies the presence of a massless particle.

0.4.2 Gauge Symmetry

One of the most important developments in particle physics, leading to the standard model, was the use of local or gauge symmetries as constructive principle. This principle is based on *local* symmetry operations, that is the elements of the symmetry group depend on space and time. It was first recognized as a powerful construction principle by Hermann Weyl in connection with a tentative extension of general relativity (Weyl (1918a); 1918b); 1919); 1923)) and then applied to quantum physicsWeyl (1929).[12] The prehistory of the gauge principle dates back to the development of classical electrodynamics in the 19th century, see (Jackson and Okun (2001)). We shall go immediately *medias in res* and present it in the final form as developed by Yang and Mills (1954).

We start with an (exact) internal symmetry having a compact semisimple Lie group $\mathcal{G}$ as symmetry group and its unitary irreducible representation ω, see (20). This representation provides also the corresponding representation of the Lie algebra of $\mathcal{G}$ by hermitian[13] generators $t^a \in \mathrm{Mat(n, C), a} = 1, \cdots, \mathrm{L} := \dim \mathcal{G}$, which satisfy

$$[\, t^a, t^b \,] = i \sum_{c=1}^{L} f^{abc}\, t^c \tag{21}$$

with the real structure constants f^{abc} of the Lie group $\mathcal{G}$. Then ω can be

[12]For a review of the historical development see (Straumann (1987); O'Raifeartaigh and Straumann (2000)).

[13]We adopt this convention mostly used by physicists

given the form ,

$$\omega = \exp\big(-i \sum_{a=1}^{L} \alpha^a\, t^a \big)\,, \tag{22}$$

with real parameters α^a.

For the non-Abelian groups $\mathcal{G} = SU(2), SU(3)$ in their respective self-representation, also called fundamental representation, the generators can be chosen in the form $t^a = \frac{1}{2}\,\sigma^a$, $a = 1, 2, 3$, and $t^a = \frac{1}{2}\,\lambda^a$, $a = 1, \cdots, 8,$, respectively, where σ^a are the Pauli matrices and λ^a the Gell-Mann matrices. The representation of the Abelian group $\mathcal{G} = U(1)$ is contained in the formulae above and in those to follow by setting $L = 1, n = 1, t^1 = 1, f^{abc} = 0$.

The symmetry is promoted to a *local* one or *gauge symmetry* by admitting the parameters α^a in (22) to depend on space-time, $\alpha^a = \alpha^a(x)$, and hence $\omega = \omega(x)$. Moreover, due to the unitary representation, $\omega(x)^* = \omega(x)^{-1}$. To account for the local representation the derivation operation has to be modified, introducing the Lie algebra-valued gauge field

$$A_\mu(x) = \sum_{a=1}^{L} A^a_\mu(x)\, t^a\,, \tag{23}$$

formed with an L-tuple of real covector fields $A^a_\mu(x)$, and which transforms in conjunction with the matter field $\psi(x)$ as

$$\psi'(x) := \omega(x)\,\psi(x)\,, \quad A'_\mu(x) := \omega(x) A_\mu(x)\,\omega(x)^* + i\partial_\mu \omega(x)\cdot\omega(x)^*\,. \tag{24}$$

Then, the *covariant derivative*

$$\nabla^A_\mu = \partial_\mu + i A_\mu(x) \tag{25}$$

transforms covariantly under the local gauge transformation (24),

$$\nabla^{A'}_\mu \psi'(x) = \omega(x) \nabla^A_\mu\, \psi(x)\,. \tag{26}$$

Furthermore, the field tensor

$$F_{\mu\nu}(x) := \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x) - i\,[A_\mu(x), A_\nu(x)] \;=\; \sum_{a=1}^{L} F^a_{\mu\nu}(x)\, t^a\,, \tag{27}$$

with components

$$F^a_{\mu\nu} = \partial_\nu A^a_\mu - \partial_\mu A^a_\nu + \sum_{b,c=1}^{L} f^{\,bca}\, A^b_\mu\, A^c_\nu \tag{28}$$

transforms under (24) homogeneously,

$$F'_{\mu\nu}(x) = \omega(x)\, F_{\mu\nu}(x)\, \omega(x)^* \,. \tag{29}$$

From (29),(28) follows that in any irreducible representation $\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu}$ is a gauge invariant Lorentz scalar, at most quadratic in the derivatives. In addition, given the groups considered, traces in different irreducible representations are proportional. In the respective self-representation of these groups we have $\mathrm{Tr}(t^a t^b) = \frac{1}{2}\, \delta^{a\, b}$, hence

$$\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu}\,|_{s.r.} = \tfrac{1}{2} \sum_{a=1}^{L} F^a_{\mu\nu} F^{a\mu\nu} \,. \tag{30}$$

We notice, that this invariant involves terms quadratic, in the non-Abelian case also cubic and quartic, in the gauge field A^a_μ, the quadratic part formed by derivatives only. For a matter multiplet ψ transforming according to a given irreducible representation a gauge invariant Lagrangian can be obtained replacing in the Lagrangian of the free field ψ the derivative by the related covariant derivative, (25), – called *minimal coupling* – and adding the invariant (30). Thus, in the case of a Dirac spinor multiplet ψ, we obtain

$$\mathcal{L}_{inv} = -\frac{1}{2g^2}\, \mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu}\,|_{s.r.} + \overline{\psi}(i\gamma^\mu \nabla^A_\mu - m)\psi \,. \tag{31}$$

As the gauge field transforms inhomogeneously, (24), a mass term $\mathrm{Tr}\, A_\mu A^\mu$ is not invariant and thus excluded. The dimensionless real parameter g acts as a coupling constant. Rescaling finally the gauge field,

$$A^a_\mu(x) \to g\, A^a_\mu(x), \tag{32}$$

in (31) the coupling constant is shifted in front of the interaction terms. We observe, that in the case of the Abelian group $\mathcal{G} = U(1)$ the Lagrangian (31) is the familiar one of spinor electrodynamics. If $\mathcal{G} = SU(3)$ and the spinor field transforming according to the self-representation of this group, (31) is the Lagrangian of the fundamental theory of strong interactions, called quantum chromodynamics (QCD), see sect. 0.5.2. There, the spinor fields, the gauge fields and the internal space are called “quarks”, “gluons” and space of “colour”, respectively.

In the conversion of the classical theory into a perturbative quantum field theory the quadratic part of the Lagrangian is supposed to determine the corresponding free quantum field theory with its propagators. This part of (31), after rescaling (32), reads

$$\mathcal{L}_2 = -\frac{1}{4}\sum_{a=1}^{L}(\partial_\nu A^a_\mu - \partial_\mu A^a_\nu)(\partial^\nu A^{a\mu} - \partial^\mu A^{a\nu}) + \overline{\psi}(i\gamma^\mu\partial_\mu - m)\psi\,. \tag{33}$$

The part $\mathcal{L}_2(A)$ resulting from the gauge field, however, is degenerate, since not all components of this field are dynamical degrees of freedom, because of the gauge symmetry. The restriction of the gauge field A_μ to its genuinely dynamical components by imposing a particular condition to eliminate the gauge freedom (as e.g. the Coulomb gauge) destroys the manifest covariance of the formulation. Such a formulation, however, appears to be instrumental in order to develop a renormalized perturbation theory.
The choice of a covariant gauge fixing condition as, after rescaling,

$$\mathcal{L}_{g.f.} = -\,\frac{1}{\alpha}\,\mathrm{Tr}\,(\partial^\mu A_\mu)^2\,|_{s.r.} = -\,\frac{1}{2\,\alpha}\sum_{a=1}^{L}(\partial^\mu A^a_\mu)^2\,. \tag{34}$$

with $\alpha > 0$, is sufficient in the case of $U(1)$, i.e. QED, but it proves to be deficient in the non-Abelian case, however. There, it leads to a S-matrix which violates unitarity, first noticed by Feynman (1963). As shown by Faddeev and Slavnov (1991), the proper choice of a gauge fixing condition as (34) is connected with the appearance of a related functional determinant. This latter *nonlocal* object can be added to the Lagrangian density in the form of a local term, employing “ghost fields” $c^a(x)$ and “antighost fields” $\overline{c}^a(x)$, $a = 1, ..., L$, transforming according to the adjoint representation of $\mathcal{G}$,

$$\mathcal{L}_{ghost} = i\sum_{a,b=1}^{L}\overline{c}^a(x)\,\partial^\mu(\partial_\mu\,\delta^{ab} - g\sum_{l=1}^{L} f^{\,lab}A^l_\mu(x))c^b(x)\,. \tag{35}$$

These new unphysical fields are scalar quantum fields which are required to satisfy the *anti*commutation relation in (7), thus not conforming with the connection between spin and statistics. Then the (classical) total Lagrangian density

$$\mathcal{L}_{tot} \;=\; \mathcal{L}_{inv} + \mathcal{L}_{g.f.} + \mathcal{L}_{ghost} \tag{36}$$

has relativistically invariant gauge fixing built in and owns a quadratic part which is non-degenerate. Thus it has the appropriate shape to serve as a point of departure in developing a corresponding relativistic quantum field theory in the form of a renormalized perturbation theory. The perturbation expansion with the renormalization procedure connected is performed in terms of Green functions. These functions are not gauge invariant individually and depend on the gauge fixing chosen. Insisting on a locally gauge invariant quantum field theory requires the Green functions to satisfy the system of Slavnov-Taylor identities (Faddeev and Slavnov (1991); Taylor (1971)), analogous to the Ward identities (Ward (1950); Takahashi (1957)) in quantum electrodynamics. Actually there exists a symmetry of the total Lagrangian (36), providing an alternative derivation of the Slavnov-Taylor identities: $\mathcal{L}_{tot}$ is invariant under the Becchi-Rouet-Stora-Tyutin (BRST) transformations, (Becchi *et al.* (1976); Tyutin (1975)), which affect all fields entering it. The state space resulting from the quadratic part of $\mathcal{L}_{tot}$ has an indefinite metric. But a space of physical states can be constructed as a quotient space on which the S-matrix fulfills perturbative unitarity. We shortly outline the construction: Subsidiary conditions in terms of charge operators due to the BRST symmetry and (the form of) $\mathcal{L}_{ghost}$ determine a subspace $\mathcal{V}_{phys}$ with positive semidefinite norm, which is spanned by physical state vectors having positive norm and (unphysical) null-vectors. The latter ones are orthogonal on the physical state vectors and form a subspace $\mathcal{V}_0 \subset \mathcal{V}_{phys}$. The subspace $\mathcal{V}_{phys}$ remains invariant under time translations, and the quotient space $\mathcal{V}_{phys}/\mathcal{V}_0$ forms the above mentioned space of physical states, (Faddeev and Slavnov (1991)).

If a quantum field carries an electric charge, its relation to observed particles turns out to be distinctly more intricate than in the case of a neutral field. The predictions of a massive quantum field theory are recorded in its S-matrix with regard to asymptotic particle states, see (17). This concept, however, has to be extended if the electromagnetic interaction is involved. Since the gauge field is massless, two types of singularities in the infrared would arise in following straight the path of renormalized perturbation theory: 1) In a Feynman integral, the integration over a "virtual" photon momentum diverges at vanishing momentum value. 2) The emission probability of a (real) photon, calculated via the corresponding S-matrixelement, diverges with vanishing frequency. These consequences point to an inherent deficiency in treating charged fields like neutral ones in establishing the renormalization process. Within perturbation theory the arising obstacles can be circumvented by using a fictitious (small) photon mass at an intermediate stage. Regarding experiments, all measurements have necessarily a limited energy and momentum resolution. Therefore, the requirement of a discrete value for m^2 in the relation (1) can never be checked exactly. Taking explicitly into account these limiting observational conditions the construction of a renormalized perturbation theory can be extended to cover the pecularities of the electromagnetic interaction in the infrared, too. To create an intermediate infrared regularization, a particular gauge fixing term (the Stueckelberg gauge) of the gauge field A_μ is introduced, as well as a fictitious mass term for this field, see (Itzykson and Zuber (1980)). Hereof results a propagator for A_μ, which is finite at vanishing momentum, allows renormalization and has a proper zero mass limit. To determine then the transition probability for a certain reaction in a fixed order of the perturbation theory, given the resolution ΔE of the counter, the respective transition probabilities of all reactions which involve additional "massive" soft photons of energies smaller than ΔE in the initial and final state have to be summed. Then, in this sum the fictitious photon mass can be made to vanish, yielding in the fixed order considered a finite observable quantity, depending on ΔE, see (Jauch and Rohrlich (1980)).[14] In short, by this constructive procedure the infrared divergences emerging from real and virtual soft photons compensate each other in a controlled way.

Notwithstanding the empirical success of this calculational procedure various attempts towards a genuine formulation of a charged state have been made, see (Morchio and Strocchi (1986)). Already in the early period of QED Bloch and Nordsieck (1937) devised a simplified model to deal with the electromagnetic interaction of a spinor field in the low-frequency domain without taking recourse to perturbation theory, see also (Bogoliubov and Shirkov (1959)). In this model the complete 2-point function of the charged field can be evaluated exactly and shows in momentum space a pole singularity, however multiplied by a further singular factor. A power series expansion of the latter one in the fine structure constant α leads to powers of the logarithm of the pole term. Later on, in the fifties, an analogous singularity structure has been derived via the renormalization group of QED, (Bogoliubov and Shirkov (1959)). As a consequence charged fields do not have a particle interpretation according to the LSZ reduction formula (17). In an imaginative language the charged particle may be described as accompagnied by a cloud of soft photons. More recently, approaching the infrared problem from first principles within the algebraic setting of Local Quantum Physics,

[14] The effective expansion parameter is $\alpha \log(\Delta E/m)$, where m is the mass of the charged particle.

in (Fröhlich *et al.* (1979)) the representations of the algebra of the asymptotic electromagnetic field in scattering states are investigated and the connection with the notion of an “infraparticle”, (Schroer (1963)), is shown. Furthermore, a unified particle concept covering the infrared problem of charged particles has been developed, (Buchholz *et al.* (1991)). A novel construction of renormalized perturbative QED which fully deals with a massless photon field as basic ingredient has been developed by Steinmann, (Steinmann (2000)). This approach, which overcomes the ultraviolet and the infrared problem jointly, makes use of elements from general quantum field theory as well as from the framework of local observables. It aims directly at the inclusive cross sections of experiments, avoiding completely to introduce an S-matrix and any intermediate infrared regularization. Instead, a “particle probe” is defined, which plays the role of a local observable detecting the emergence of particle-like objects.

0.4.3 Lattice Gauge Theories

A great advance in the method of lattice regularization was achieved by the construction of locally gauge invariant lattice theories. The first step was made in statistical mechanics, where Wegner (1971) realized that for the construction of gauge invariant quantities on has to attach dynamical variables not only to the points of a lattice, but also to the links between the points. Wilson (1974) constructed a lattice version of an Euclidean relativistic non-Abelian gauge field theory with spinor fields.

In this formulation to each ordered link between neighbouring lattice points x_i, $x_j = x_i + a\, e^{(\mu)}, e^{(\mu)}$ the unit vector in direction μ, a the lattice spacing there is attached a copy of the gauge group $\mathcal{G} = SU(3)$ in its self-representation, i.e a unitary matrix $u(x_i, x_j)$ such that $u(x_j, x_i) = u(x_i, x_j)^*$. Furthermore, to each lattice point is attached a fermion field $\psi(x_i)$. Then, the lattice action of the pure gauge part is

$$S_{YM} = \frac{1}{g^2}\,\mathrm{Re} \sum \mathrm{Tr}\Big(1 - u(x_i, x_j)u(x_j, x_k)u(x_k, x_l)u(x_l, x_i)\Big)\,, \tag{37}$$

where x_i, x_j, x_k, x_l are the four corner points of an elementary lattice square, called “plaquette”, and the sum extends over all plaquettes. A simplified version[15]) of the fermionic lattice action reads

$$S_F = a^3 \sum \overline{\psi}(x_i)\gamma_\mu u(x_i, x_j)\psi(x_j) + a^4 m \sum \overline{\psi}(x_i)\psi(x_i)\,, \tag{38}$$

where the first sum extends over the links, affecting also the related γ_μ, and the second one over the sites of the lattice. Finally, a local gauge transformation is an independent map of each lattice site $x_i \to \omega(x_i)$, the self-representation of $\mathcal{G}$, together with the replacements

$$u(x_i, x_j) \to \omega(x_i)\, u(x_i, x_j)\, \omega(x_j)^*, \quad \psi(x_i) \to \omega(x_i)\, \psi(x_i)\,, \tag{39}$$

which obviously leave S_{YM} and S_F invariant. Heuristically, in a formal continuum limit $a \to 0$, suggested from the continuum parallel transporter[16]

$$u(x_i, x_j) = \mathcal{P}\, \exp i \int_{x_i}^{x_j} A_\mu dx_\mu \;\to \exp i a A_\mu(x_i)\,, \tag{40}$$

provides $S_{YM} + S_F \to \int d^4x\, \mathcal{L}_{inv,E}$, where $\mathcal{L}_{inv,E}$ is the Euklidean version of (31).

This lattice regularization leads to well defined correlation functions (Schwinger functions, see sect. 0.3.1) and avoids the introduction of "unphysical" degrees of freedom, like ghost fields (see (35)). The still unachieved task is however to prove the existence of a physically sensible continuum limit $a \to 0$.

Realistic lattice gauge theories are analytically inaccessible, but the immense progress in computing technology has made it possible to perform numerical calculations of some Schwinger functions on fairly extended hypercubic lattices (up to 64^4 lattice points). The best studied part is pure gauge theory. The inclusion of fermions poses in principle no problems, since the fermion integration can be performed analytically. The resulting determinant however exhibits long distance correlations which make the numerical calculations of Schwinger functions impossible, at least at the present state of computing technology. Therefore calculations, especially with light fermions have to be based on additional assumptions. The lattice version of non-Abelian gauge theory was constructed as a model to perform numerical calculations in QCD (Wilson (1974)), the field theory of strong interactions, we therefore come back to its achievements in the specific section 0.5.2.

[15]For the improved form see (Seiler (1982)

[16]$\mathcal{P}$ denotes path ordering

0.4.4 Spontaneous Symmetry Breaking: Gauge Theories

The important role of gauge symmetry in QED led early (Klein (1938)) to consider gauge symmetries in strong and weak interactions, too. The success of the non-renormalizable four-fermion coupling (Fermi (1934)) in weak interactions made it clear that a description by the exchange of bosons would imply a very large mass of the latter. Therefore, an underlying gauge symmetry had to be strongly broken, since gauge bosons in an unbroken symmetry are massless. An explicit symmetry breaking, (Bludman (1958)), by introducing a mass term, would render the theory non-renormalizable again. At first sight, a mechanism of spontaneous symmetry breaking also seems to be inappropriate, since because of the Goldstone theorem spontaneous symmetry breaking of a continuous symmetry is accompagnied by massless bosons. This unwanted linkage, however, can be circumvented. Within a *local gauge theory*, spontaneous symmetry breaking without Goldstone bosons has been achieved, (Higgs (1964); Englert and Brout (1964); Guralnik *et al.* (1964)), termed the *Higgs mechanism*. Eventually, the standard model of elementary particles as a whole emerged as a local gauge theory. In particular, the combined electromagnetic and weak interaction with the local gauge group $U(1) \times SU(2)$ is based on the Higgs mechanism in the weak sector.

We exemplify this mechanism in the simpler Abelian Higgs model. In its classical version a complex scalar field $\phi(x)$ and a massless real vector field $A_\mu(x)$ interact with Lagrangian density

$$\mathcal{L}_{inv}(\phi, A) = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(\partial_\mu - igA_\mu)\overline{\phi}\cdot(\partial^\mu + igA^\mu)\phi - \lambda\,(|\phi|^2 - \rho^2)^2\,, \quad (41)$$

where $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$ and g, λ, ρ are real positive constants.
The Lagrangian density $\mathcal{L}_{inv}$ is invariant under *local gauge transformations*

$$A_\mu(x) \to A_\mu(x) + \partial_\mu\zeta(x)\,, \quad \phi(x) \to \exp(-ig\zeta(x))\cdot\phi(x)\,, \qquad (42)$$

with a real function $\zeta(x)$, cp.(24). Obviously, this invariance holds a fortiori for a constant ζ (*global gauge invariance*). The energy density resulting from the Lagrangian (41) has a degenerate minimum formed by the set of configurations $\{A_\mu(x) \equiv 0, \phi(x) = \rho\exp i\gamma\,|\,0 \le \gamma < 2\pi\}$. To shape hereof a *perturbative* quantum field theory, a particular configuration in the degenerate minimum is arbitrarily selected, say by choosing $\phi_{min} = \rho > 0$, and restricting the scalar field to have the form

$$\phi(x) = \rho + \varphi(x), \quad \varphi(x) = \varphi_1(x) + i\varphi_2(x)\,, \quad \varphi_1, \varphi_2 \text{ real}, \qquad (43)$$

with $\varphi(x)$ vanishing for $|x| \to \infty$. As a consequence the global $U(1)$-symmetry present in (41) is now broken, the local gauge invariance (42),however, still holds, if the gauge functions $\zeta(x)$ are required to vanish asymptotically.

Using (43), the Lagrangian density (41) takes the form

$$\mathcal{L}_{inv}(\rho + \varphi, A) = \tilde{\mathcal{L}}_2(\varphi, A) + \tilde{\mathcal{L}}_{int}(\varphi, A)\,, \tag{44}$$

with quadratic part

$$\begin{aligned} \tilde{\mathcal{L}}_2(\varphi, A) = & - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(g\rho)^2 A_\mu A^\mu + g\rho\, A^\mu\, \partial_\mu\varphi_2 \\ & + \tfrac{1}{2}(\partial_\mu\varphi_1)\partial^\mu\varphi_1 - \tfrac{1}{2}(8\lambda\rho^2)\varphi_1^2 + \tfrac{1}{2}(\partial_\mu\varphi_2)\partial^\mu\varphi_2\,. \end{aligned} \tag{45}$$

From $\tilde{\mathcal{L}}_2(\varphi, A)$ result the propagators in perturbative quantum field theory. Since (44) is still locally gauge invariant, fixing the gauge degrees of freedom is required. At first glance, requiring $\varphi_2 \equiv 0$ appears appealing (*unitary gauge*): from (45) we read off, that the originally massless vector field became massive with a mass $m = g\rho$, hence acquiring a longitudinal degree of freedom, whereas the originally complex scalar field now is real with mass $M = 2\rho\sqrt{2\lambda}$. Determining the propagators, however, one encounters a bad ultraviolett-behaviour in case of the vector field, which does not allow a renormalizable perturbation expansion.
The t'Hooft gauge fixing adds to (44), and thus to (45) the term

$$\mathcal{L}_{g.f.} = -\frac{1}{2\alpha}(\partial_\mu A^\mu - \alpha g\rho\varphi_2)^2\,, \quad \alpha \in \mathbf{R}_+\,, \tag{46}$$

which is an extension of (34). Thus, the new quadratic part is given by

$$\begin{aligned} \tilde{\mathcal{L}}_2(\varphi, A) + \mathcal{L}_{g.f.} = & -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(g\rho)^2 A_\mu A^\mu - \tfrac{1}{2\alpha}(\partial_\mu A^\mu)^2 \\ & +\tfrac{1}{2}(\partial_\mu\varphi_1)\partial^\mu\varphi_1 - \tfrac{1}{2}(8\lambda\rho^2)\varphi_1^2 + \tfrac{1}{2}(\partial_\mu\varphi_2)\partial^\mu\varphi_2 - \tfrac{1}{2}\,\alpha(g\rho)^2\varphi_2^2\,. \end{aligned} \tag{47}$$

It is again diagonal in the fields. In addition to the fields present in (45) the scalar field φ_2 appears with mass $m\sqrt{\alpha}$, depending on the gauge parameter α. This dependence indicates the unphysical nature of the field, produced by the particular gauge fixing employed. The virtue of this choice is to lead to a propagator of the vector field having the form[17]

$$D_{\mu\nu}(k) = \Big(g_{\mu\nu} + \frac{(\alpha-1)k_\mu k_\nu}{k^2 - \alpha m^2 + i\epsilon}\Big)\frac{1}{k^2 - m^2 + i\epsilon} \tag{48}$$

[17]This form also reveals that the part $\partial_\mu A^\mu$ of the vector field is unphysical, too.

the ultraviolett-behaviour of which is compatible with perturbative renormalizability.

The Higgs mechanism generated in the standard model is built on non-Abelian gauge vector fields with gauge symmetry $SU(2)$ together with a $SU(2)$-doublet of complex scalar fields. Then the Lagrangian is the very analog of (41), with corresponding form of the self interaction, pure gauge term (30), and covariant derivative (25). As a consequence, the degenerate minimum of the energy density arising now corresponds to a sphere in $\mathbf{R}^4$ in place of a circle in $\mathbf{R}^2$ met before in the Abelian case. Proceeding on an analogous route as (43), the still present local $SU(2)$-gauge symmetry requires a restriction to classes of gauge-equivalent field configurations, i.e. gauge fixing. In contrast to the Abelian case, however, requiring in the non-Abelian theory a relativistically invariant gauge fixing analogous to (46), the related Lagrangian $\mathcal{L}_{ghost}$ has to be introduced in addition, similarly as in (34), (35). As the analogue of (46) involves scalar fields, $\mathcal{L}_{ghost}$ contains interaction terms of ghost/antighost with the scalar fields. As a consequence of the choice $\mathcal{L}_{g.f.}$ analogous to (46) there then appear three vector fields with mass m and one scalar field with mass M as the physical fields; in addition three further scalar fields as well as three ghost-antighost pairs are on stage, all with mass $m\sqrt{\alpha}$ and having the status of auxiliaries. Moreover, the propagator of the vector fields is of the form (48).

In contrast to the perturbative approach developed to cope with a fully-fledged quantum gauge field setting, gauge theories defined on a Euclidean space-time lattice as introduced by Wilson can be rigorously dealt with, i.e. without recourse to a (formal) perturbation expansion. With Minkowski space exchanged by the Euclidean lattice, local gauge symmetry can be strictly kept in the analysis, gauge fixing is not needed. Exploiting local gauge invariance of a gauge-invariant version of the XY model, Elitzur (1975) has shown, that spontaneous breaking of a local gauge symmetry is not possible without gauge fixing. This result has been sharpened and significantly extended by De Angelis, de Falco and Guerra,(De Angelis *et al.* (1978)). They considered in particular the Euclidean lattice version of the Abelian Higgs model (41) and proved, that the expectation value of the scalar field vanishes upon reducing to zero an external field initially coupled to the system; removing the external field leads to a gauge invariant state. This lattice Abelian Higgs model belongs to the family of Higgs type models investigated by Osterwalder and Seiler (1978) using convergent cluster expansions. They showed exponential clustering (also) in a special region of the theory's parameters, which can be read as a sign of dynamical mass generation. To be valid in Minkowski space, all these interesting rigorous results still have to persist in the continuum limit of the lattice (and Wick rotation).

Remarkably, a massive non-Abelian gauge model can be perturbatively constructed without resorting to the Higgs mechanism in a classical precursor theory. This has been achieved by Scharf and coworkers (Scharf (2001); Dütsch and Scharf (1999); Aste *et al.* (1999)), following the causal method of Epstein and Glaser (1973) in constructing the S-matrix of the model. In this approach massive vector bosons are introduced as fundamental entities and as a substitute of the BRST symmetry the requirement of causal gauge invariance is imposed via the asymptotic fields. In the perturbative inductive construction of a (re)normalizable theory then the need for scalar fields arises, and the form of the interaction is determined.

0.5 The Standard Model of Particle Physics

0.5.1 Field Content and Interactions

The standard model of particle physics is the renormalizable quantum field theory of subatomic particles. It describes successfully the strong, the electromagnetic and the weak interaction. In the following we shall give a very short outline of its results and methods, which reflect the facet-like character of quantum field theory in general.[18] Most of the facets contribute to the successes of the theory, but on the other hand the fragmentation of the methods leaves some fundamental questions open.

The basic fermion fields entering the Lagrangian are those of the quarks and leptons, the interaction is determined by minimal gauge invariant interaction leading to the gauge-vector fields. The gauged groups are $SU(3)$ for the strong interaction of the quark fields and the product $U(1) \times SU(2)$ for the electromagnetic and weak interaction.

A remarkable feature of the standard model is the absence of the observed hadrons among the fundamental fields, whereas no asymptotic particles correspond to the fundamental hadronic fermion (quark) fields of the theory. In this respect the unified field theory of Heisenberg and coworkers (see Dürr *et*

[18]Concise reviews on the principal aspects of the Standard Model can be found in the Review of Particle Physics (Nakurama, K. *et al.* (2010))

al. (1959)), a theory based on fundamental fermion fields can be considered as a precursor.

Whereas the leptons are only subject to the electroweak interaction, the quark fields occur both in the electroweak and strong interaction sector. We therefore give first a short description of this more limited sector.

0.5.2 Quantum Chromodynamics

Quantum chromodynamics (QCD) is generally accepted as the microscopic theory of strong interactions. The gauge group of QCD is SU(3), the so called colour group (Han and Nambu (1965); Fritzsch *et al.* (1973)); the quark fields transform under the fundamental representation of that group, they are thus triplets. There are 8 gauge vector fields (gluons), corresponding to the eight generators of SU(3). The Lagrangian is given by (31). There are 6 triplets of quark fields, distinguished by different quantum numbers, called flavours, and with different renormalized mass. Since quarks do not occur as asymptotic particles, there is no on-mass-shell definition of the renormalized mass, it is therefore an entirely internal parameter. The renormalized quark masses vary from a few MeV (*up* and *down* quark) to 171 GeV (*top* quark).

An important feature of QCD is the asymptotic freedom (Gross and Wilczek (1973); Politzer (1973)): The value of the gauge coupling decreases with increasing mass of the subtraction point. From the fundamental point this may be an important advantage, since it indicates that it might be a consistent theory with a Borel-summable perturbative series (see end of sect. 0.3.3). Since the value, where the gauge coupling $\alpha_s = g_s^2/(4\pi)$ is small, is far above the Compton wave length of ordinary hadrons, it is impossible to apply perturbation theory to calculate such important quantities as hadron spectra and hadronic cross sections. In order to apply perturbation theory at all, one has to look for special processes and try to separate the long distance behaviour from the short distance behaviour, to which perturbation theory can be applied. For different processes this is possible with varying degrees of rigour. The most promising method is the operator product expansion or short distance expansion of Wilson (Wilson (1969); 1971); Wilson and Zimmermann (1972); Zimmermann (1973a); 1973b)). A product of operators is expanded in the form:

$$\phi_A(x)\phi_B(y) \overset{x\to y}{\sim} \sum_N C_N(x-y)\; O_N\Big(\frac{x+y}{2}\Big) \tag{49}$$

For a free theory the singular behaviour of the coefficient function C_N is determined by the dimensions of the operator. In perturbation theory logarithmic deviations can be calculated. Making full use of the machineries of perturbation theory in this way e.g. the dependence of cross sections for scattering of leptons on protons have been calculated.[19] More precisely, if the dependence of the cross sections on the total hadronic energy W is known for $Q = \sqrt{(p'_\ell - p_\ell)^2}$, the fixed (large) momentum transfer to the leptons, one can predict the dependence on W for fixed higher values of Q (see e.g. Altarelli (1982)). Similar calculations have been performed in electron-positron annihilations at large energies. The agreement between experiment and theory is satisfactory. Unfortunately perturbation theory in strong interactions is less apt to explain striking phenomena, but rather special experiments are invented in order to test the theory.

The most salient feature of particle physics, the profusion of "elementary particles", in the sense of definition 2 in sect. 0.2.1, cannot be treated in perturbation theory. Here many efforts have been made by QCD inspired models, some with remarkable phenomenological success, especially QCD-inspired quark potential models. In this paper we do not discuss, however, these more particular approaches.

The only approach to hadron spectroscopy which starts directly from the Lagrangian is lattice gauge theory, see e.g. (Creutz (1985); Montvay and Münster (1997); Rothe (2005)). Assuming that finite lattice spacing is a good approximation to the continuum theory one has verified numerically confinement for static colour charges in a pure gauge theory: the energy increases indefinitely with increasing distance between the colour charges (static quarks). In order to calculate realistic hadron spectra, more assumptions have to be made, especially the contributions of quark-antiquark loops can only be treated approximatively. The resulting mass ratios are however in satisfactory agreement with experiment. A recent comprehensive survey of the basic techniques of numerical lattice QCD with emphasis on their theoretical foundations is given in (Lüscher (2010)).

---

[19] Here the short distance expansion, $x \to y$ has to be extended to the light cone expansion $(x - y)^2 \to 0$

0.5.3 The Electroweak Interaction

All fermion fields – quarks and leptons – are subject to the electroweak interaction. Here the gauge group is $U(1) \times SU(2)$. All left handed fermions[20] are grouped into doublets transforming under the $SU(2)$ part of the symmetry (weak isospin), the right handed fermions transform as singlets under $SU(2)$. The lower component of a lepton doublet is a negatively charged left handed lepton (e^-, μ^- or τ^-), the upper one the corresponding neutrino.[21] The upper component of a doublet of left handed quarks consists of a quark field with electric charge $+\frac{2}{3}$ (u, c or t quark), the lower component consists of a linear combination[22] of quarks with charge $-\frac{1}{3}$ (d, s or b quark).

As the gauge group $U(1) \times SU(2)$ is not simple, an Abelian gauge field $B_\mu(x)$, corresponding to $U(1)$, and a non-Abelian gauge field (see (23)) $W_\mu(x) = \frac{1}{2}\sum_{a=1}^3 \sigma^a \, W_\mu^a(x)$, corresponding to $SU(2)$, appear.

A hypercharge Y is introduced, which governs the coupling to the $U(1)$ gauge field B_μ. The electric charge is given by $Q = T_3 + Y/2$, where T_3 is three-component of the weak isospin.

The Lagrangian has the form (31) with rescaling the gauge fields, see (32). The covariant derivatives are:

$$\text{for the doublet :} \quad \partial_\mu \mathbf{1} - i\frac{g'Y}{2} B_\mu \mathbf{1} + ig\, W_\mu \tag{50}$$

$$\text{for the singlet :} \quad \partial_\mu - i\frac{g'Y}{2} B_\mu . \tag{51}$$

One starts with a classical Lagrangian without fermion masses. In order to generate masses, an additional scalar field is introduced, the so called *Higgs field*. In the standard model it transforms as a doublet under the $SU(2)$ part and has hypercharge $Y = +1$. The classical Lagrangian for that field is analogous to the second and third term in (41) with the covariant derivative (50). It leads to a spontaneous breaking of the symmetry, as discussed in more detail in sect. 0.4.4. The unitary gauge is chosen in such a way that only the expectation value of the lower, the electrically neutral,

[20]For a left handed spinor field the spin polarization is antiparallel to the momentum, for a right handed one it is parallel.

[21]Neutrino oscillations indicate small neutrino masses, this has to lead to some modifications of the Standard model. Given the preliminary state of the field, we shall not discuss the possible consequences

[22]The coefficients of the linear combinations are the CKM matrix

component of the Higgs doublet is different from zero. This has as a consequence that charged gauge bosons and the difference of the $U(1)$ and of the third isospin component of the $SU(2)$ gauge field, $Z^0_\mu \sim g' B_\mu - g\, W^3_\mu$ acquire a mass term. The sum $A_\mu \sim g' B_\mu + g W^3_\mu$ remains massless, it is the electromagnetic field. The now massive boson fields $W^\pm_\mu$ and Z^0_μ are the intermediate boson fields mediating the weak interaction. The charged intermediate bosons lead in the limit of large mass to the four-fermion coupling, introduced by Fermi (1934), whereas the Z^0 boson induces neutral currents which in turn give e.g. rise to electron-neutrino scattering. Observation of those processes made allowed a prediction of the masses of the intermediate bosons from the classical Lagrangian with an accuracy of a few percent. This in turn allowed the experimental observation in dedicated experiments. Very precise measurements of observables in weak interactions and an analysis of the perturbative corrections allowed the prediction of an up to then unobserved[23] very heavy quark, the top quark.

From its influence due to its presence in internal loops in electroweak perturbation theory a renormalized mass of about 170 GeV/c^2 was predicted, the observed mass deduced from high energy reactions turned out to be very near this value (171 GeV/c^2). This successful and precise prediction of a new quark is certainly a highlight of perturbation theory.

The quark masses are generated by couplings of the Higgs boson to the (massless) quarks, they are proportional to the nonvanishing expectation value of the Higgs field. The couplings are adjusted in order to yield the observed renormalized quark masses.

The purely fermionic sector of electroweak interactions is plagued by quantum corrections which break the symmetry (anomalies) and thus jeopardize the renormalizability of the theory. But fortunately corresponding anomalies occur in the quark sector with opposite signs such that the standard model with three doublets of lepton fields and the same number of quark fields is free of anomalies, (Bouchiat *et al.* (1972); Faddeev and Slavnov (1991)).

---

[23] Since quarks are not asymptotically observable particles, *observed* has a somewhat indirect meaning here

0.6 Effective Theories

0.6.1 Decoupling and Non-Decoupling Theories

Effective field theories, as a special approach to (perturbative) quantum field theory have received much attention in the last decades (see e.g. Weinberg (2009) for a recent short account). As will be stressed in the final section, a symbolic construction leads inherently to a theory which can be called *effective theory*. We therefore in our context do not consider effective theories as a proper facet of quantum field theory, but rather as an approach to perturbation theory which illustrates our point of view.

There are two types of effective theories, those which show a decoupling of the renormalizable from the effective part, and those which show no such decoupling and are not renormalizable.

A *decoupling effective theory* is a simplification of a renormalizable theory containing both light-mass and heavy-mass degrees of freedom (fields with small and large renormalized mass m and M respectively). Considering only correlation functions of the light-mass fields, it is intuitive that at low energies the heavy-mass fields play no role. Indeed it has been shown (Appelquist and Carrazone (1975); Kim (1995)) that one can obtain any correlation function of light-mass fields to any order from the (renormalizable) Lagrangian containing only the light-mass fields and additional insertions. The insertions are called irrelevant, that is their contributions are suppressed by powers of $(\frac{m}{M})$. A well known example of this type is a theory of leptons (i.e. an extension of QED for electrons) as effective theory of the electroweak interactions (see sect. 0.5.3). Here the renormalizable part of the theory is the QED Lagrangian plus a free Lagrangian for neutrinos, the lowest dimensional insertion is the four fermion interaction, which in turn can be obtained from the full electroweak theory. In a *non-decoupling effective theory* one has interaction terms which may leed to non-renormalizable theories, that is for each order in the expansion new counterterms may have to be introduced to balance ultraviolet divergences. A time honoured example is Fermi's theory of weak interactions (Fermi (1934)) if its four-fermion coupling is not treated as an insertion but as a genuine interaction term. An attempt to get insight in the confinement mechanism is based on the effective Lagrangian of an extended $\mathcal{N} = 2$ supersymmetric gauge theory, (Seiberg and Witten (1994)). This effective Lagrangian exhibits confinement of fermions through Abrikosov strings, the mechanism conjectured in the t'Hooft-Mandelstam model (Mandelstam (1976); 't Hooft (1978)) of QCD. An example of an effective theory, which is directly related to phenomena, will be discussed in the next subsection.

0.6.2 Spontaneously Broken Chiral Symmetry

Among the strongly interacting particles, the pions have by far the smallest mass. Therefore it was conjectured that they are the Goldstone bosons of a spontaneously broken symmetry, see sect. 0.4.1.[24] The observation that the pions form an isovector of pseudoscalar particles led to the conjecture that the underlying Lagrangian is invariant under the group $\mathcal{G} = SU(2) \times SU(2)$, but that it is spontaneously broken to a group $\mathcal{H} = SU(2)$. This led to the construction of an effective Lagrangian reflecting these symmetry properties. One starts with the conjecture: For a given set of asymptotic states with the most general Lagrangian allowed by the assumed symmetries yields the most general S-matrix elements consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetries. Weinberg (1979) calls this conjecture "a theorem which has never been proven". Leutwyler (1994) stresses the equivalence with the underlying theory, in this case QCD, and points out the importance of off-shell elements.

As mentioned in sect 0.4.1 the Goldstone-boson fields are in the coset space $\mathcal{G}/\mathcal{H}$. A convenient parametrization for the Goldstone fields is given by the $SU(2)$ matrix:

$$U(\Phi) = \exp[\frac{i\sqrt{2}}{f_\pi}\Phi] \tag{52}$$

Here $\Phi = \frac{1}{\sqrt{2}}\vec{\sigma}\vec{\phi}$, with $\vec{\phi}$ the triplet of pion fields and f_π the pion decay constant, $\vec{\sigma}$ are the Pauli-matrices.

A non-polynomial Lagrangian[25] incorporating all symmetry properties is constructed as a trace function of $U(\Phi)$ and $\partial_\mu U(\Phi)$. The contribution with lowest mass dimension is

$$\mathcal{L}_2 = \frac{f_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U(\Phi)^\dagger \partial^\mu U(\Phi)\right) \tag{53}$$

[24]The fact that the mass is not completely zero is assumed to be a consequence of an additional direct symmetry breaking. Therefore the pions are sometimes called *pseudo-Goldstone bosons*

[25]There are also linear realizations of chiral symmetry (Schwinger (1957); Gell-Mann and Levy (1960)) but they do not lead to phenomenologically satisfactory results

$$= \frac{1}{2}\mathrm{Tr}\Big(\partial_\mu\Phi\,\partial^\mu\Phi\Big) + \frac{1}{12f_\pi^2}\mathrm{Tr}\Big(\left(\Phi\,\partial_\mu\Phi\; - \partial_\mu\Phi\,\Phi\right)\left(\Phi\,\partial^\mu\Phi - \partial^\mu\Phi\,\Phi\right)\Big)$$
$$+O\left(\frac{\Phi^6}{f_\pi^4}\right). \qquad (54)$$

The first term on the left hand side is the free Lagrangian for the massless pion fields, the second and higher terms in the pion field $\vec{\phi}$ describe the interaction.

The perturbative expansion is in the order of derivatives (momenta) of the pion field. Higher order terms lead to divergent loops which have to be renormalized, that is additional terms have to be introduced. The perturbation theory is well defined to each order in momentum, but with the increase of powers of momenta the number of additional terms increases. To the Lagrangian one can add interactions with external fields like the isospin doublet of nucleons, the electromagnetic and weak current as well as symmetry breaking terms.

This development started before QCD was generally accepted as the quantum field theory of strong interactions and it even was, by its strong relation to the method of current algebra, one of the roots of the Standard Model. Interesting short accounts of the history of spontaneous symmetry breaking and its use in particle physics are given by the protagonists of this field, Y. Nambu in (Hoddeson and others (1997), p. 510ff) and S. Weinberg, in the same volume, p. 36ff.

We now want to show in which sense the chiral theory can be considered as an effective theory for certain aspects of a microscopic, renormalizable one, namely QCD.

The fermionic sector of the classical QCD Lagrangian with N_f massless quark fields can be written in terms of the left and right handed Weyl-spinor fields ψ_L^j and ψ_R^j for the quark fields:[26]

$$\mathcal{L}_{\text{quark}} = \sum_{j=1}^{N_f}\Big(\bar{\psi}_L^j i\gamma^\mu(\partial_\mu + ig_s\,\tfrac{1}{2}\lambda_a A_\mu^a)\psi_L^j + \bar{\psi}_R^j i\gamma^\mu(\partial_\mu + ig_s\,\tfrac{1}{2}\lambda_a A_\mu^a)\psi_R^j\Big). \qquad (55)$$

The Lagrangian is invariant under the rotations of the global *chiral* sym-

[26] In terms of the Dirac field ψ_D, the right- and left-handed Weyl-spinor fields are given by $\psi_{L/R} = \frac{1}{2}(1 \mp \gamma_5)\psi_D$.

metry group[27] $\mathcal{G} = SU(N_f)_L \times SU(N_f)_R$

$$\psi_L \xrightarrow{\mathcal{G}} g_L\,\psi_L\,, \qquad \psi_R \xrightarrow{\mathcal{G}} g_R\,\psi_R\,, \qquad A^a_\mu \xrightarrow{\mathcal{G}} A^a_\mu\,, \qquad g_{L/R} \in SU(N_f)_{L/R}\,. \tag{56}$$

We shall concentrate here on the case $N_f = 2$, that is isospin symmetry.[28] The conserved (Noether) currents of the symmetry group $\mathcal{G}$ are

$$J^k_X = \tfrac{1}{2}\bar{\psi}_X \gamma^\mu \sigma^k \psi_X\,, \qquad X = L, R; \quad k = 1, 2, 3\,. \tag{57}$$

Though one is not in the position to calculate the hadron spectrum analytically from this Lagrangian (55), one can explain with it qualitatively several observed properties of hadron dynamics. For this purpose one has to assume that the full chiral symmetry $\mathcal{G} = SU(2)_L \times SU(2)_R$ is spontaneously broken to the symmetry $\mathcal{H} = SU(2)_V = SU(2)_{L+R}$ of the vacuum state. In principle this feature should be a consequence of the interaction.

The Noether current of the remaining unbroken symmetry $\mathcal{H}$ is

$$\{J^k_V\}_\mu = \tfrac{1}{2}\{J^k_R + J^k_L\}_\mu = \tfrac{1}{2}\bar{\psi}_D \gamma_\mu \sigma^k\,\psi_D\,. \tag{58}$$

The Goldstone boson fields correspond to the generators of the spontaneously broken symmetry, that is the charges of the axial current $J^k_A = \frac{1}{2}(J^k_R - J^k_L)$.

Therefore there exist 3 pseudoscalar Goldstone bosons which are identified with the triplet of pions. The small but finite mass of the pions is explained by a small (renormalized) mass of the light quarks in the QCD Lagrangian.

The spontaneous symmetry breaking also explains why there is no parity degeneracy of the hadronic states, this would occur for an unbroken chiral symmetry. These qualitative remarks on the consequences of spontaneous chiral symmetry breaking can be made quantitative in chiral perturbation theory, as discussed above. Quark masses in the QCD Lagrangian give rise to an explicit breaking of the chiral symmetry and hence induce finite masses for the (pseudo) Goldstone bosons.

---

[27] The additional symmetry $U(1)_L \times U(1)_R$ is not discussed here

[28] The masses of the two lightest quarks are below 10 MeV (in the $\overline{\text{MS}}$ substraction scheme at a scale of 2 GeV). The strange quark has also a definitely lower mass (< 120 MeV) than the heavy ones (> 1.2 GeV) and therefore a similar reasoning can be applied to $N_f = 3$

0.7 Epistemological and Historical Considerations

0.7.1 The Facets of Quantum Field Theory as Symbolic Constructions

It seems to be one of the great miracles, that physics is very successful in applying mathematical reasoning to explain natural phenomena, where explaining implies not (only) general classification but also very specific predictions. Mathematics is arguably the purest mental construction, whereas phenomena have certainly also constructive aspects, but the once introduced quantitative aspects are largely independent of the construction and furthermore are in many cases intimately intertwined with other phenomena. To give an example: The influence of chemical binding on the magnetic moment of a proton might seem a rather constructed phenomenological quantity, but it is also relevant for such mundane applications as magnetic resonance imaging. This seems to be an esoteric example, but experimental physics tends more and more in this direction. On the other hand also the theories become more abstract and reveal in this way their highly symbolic and constructive character. The rotation group $SO(3)$ is a mathematical construction very close to, and initially motivated by, everyday experience, but its universal covering group, $SU(2)$ is quite remote from this experience. Nevertheless these mathematical abstractions, moving away from intuitive reasoning, are essential to give a theoretical description of experimentally accessible microscopic phenomena, such as spin or isospin.

Whereas everybody should agree that theories are symbolic constructions, after all they operate with mathematical objects which per se are not related to direct experience, there seems to be disagreement about the role and uniqueness of the symbolic representation. In the 19th century a “realistic” interpretation say of electric phenomena was based on a mechanistic interpretation, that is the theory of mechanics was supposed to be the unique and adequate representation. But already at the end of that century, Helmholtz e.g. (in Hertz (1894)), preface) had a wider view of symbolic representation. He writes that other physicists, as Lord Kelvin and Maxwell, were evidently more satisfied by mechanical explanations than “by the plain most explanations of the facts and their laws as they are given in physics by the systems of differential equations”, whereas he himself felt most assured by the latter representation.

Quantum field theory comes into view in various branches, grown in the course of time, the basic concepts of which we have pointed out in an overview in sect.0.3. These branches, called “facets” by us, are at first sight theories of their own, each based on its particular intrinsic concept. Notwithstanding the apparent differences in their respective concepts these facets form particular and restricted approaches towards a common *ideal* goal, named relativistic quantum field theory. This goal can be characterized in short as follows: to transmute classical field theory on Minkowski space-time into a consistent quantum theory, which shows locality as a central property, and has phenomena of subatomic particles as its physical domain. It is a goal, since the construction of such a quantum field theory with inherent interaction has not yet been achieved on a sound mathematical footing in four space-time dimensions, even without being subject to any “external” demand posed by experiment. Each of these facets approaches autonomously certain features of the ideal goal, ignoring largely other aspects. Moreover, looking at the respective set-up of the various facets one recognizes marked differences, preventing to consider these facets as mere subtheories of a coherent embracing unit. To clarify the epistemological implications of this state of affairs deserves special attention. We put forward the view to consider these facets as *synchronous* symbolic constructions. As regards (theoretical) physics, a symbolic construction should comply with three basic requirements:
i) It is a mathematical structure created in accordance with a logically consistent general concept.
ii) A semantic interpretation of the structural relations generated by the theory formed provides a correspondence with specific observational relations in phenomena, which explains these latter ones and gives them meaning in terms of the former ones. The symbolic construction becomes empirically *testable* only via this correspondence between the structural (i.e. mathematical) and the observational relations, given by a list of prescriptions.
iii) There is a significant (limited) domain of empirical adequacy, within which the theory can provide explanation, prediction and quantitative description of specific phenomena.
This view emphasizes the constructive character of a physical theory. It stresses the essentially limited range of empirical adequacy of such a theory and the symbolic character of its basic ingredients.

Concretely, we notice, that the Wightman postulates of *General Quantum Field Theory* are widely recognized to form the core properties of a local quantum field theory, any dynamical content, however, remains hidden. As already emphasized in sect. 0.3.1, the PCT theorem and the relation between spin and statisticts are general consequences of these postulates with far reaching phenomenological consequences.

Constructive Quantum Field Theory endeavours to establish on a sound mathematical footing concrete quantum field theories with interaction. These mathematically ambitious efforts succeeded in two and three dimensional space-time; the constructions, based on functional methods, satisfy the analogue of the Wightman postulates, they demonstrate the mathematical consistency of these postulates in the case of a theory with interaction. Moreover, there are some initial steps towards a rigorous non-perturbative construction of a non-Abelian gauge theory in four space-time dimensions, (Balaban (1989a); 1989b)).

Local Quantum Physics focusses directly on observations, to be formulated within quantum theory. As all measurements and experimental preparations are performed in bounded regions of space-time, an algebra of local observables is introduced as basic structure. Central properties considered are connected with charge quantum numbers (electric, weak, flavour, etc). This approach covers charged systems without using unobservable (charged) fields. Nevertheless, the successful calculations of particular effects within renormalized perturbation theory of QED rely on local charged fields as basic constituents of the theory. It does not appear obvious how to reconstruct these achievements circumventing the use of such fields.

Characterized briefly, the three facets, General Quantum Field Theory, Constructive Quantum Field Theory, and Local Quantum Physics are more of a paradigmatic nature and do not intend to cope directly with actual phenomena.

Renormalized Perturbation Theory generates interacting quantum field theories in four space-time dimensions, also incorporating the apparently decisive local gauge symmetry, at a price to be paid, however: seen mathematically, the theory appears only as a *formal* power series in a coupling strength, disregarding questions of convergence. There are indications, that at most an asymptotically free theory may be summable. In applications of this method, the series is restricted to a few orders, believed to be justified in the case of weak interaction strength. Within this approach specific physically sensible models have been constructed, culminating in the creation of the Standard Model. As regards the above mentioned item iii) of a symbolic construction, this facet is the most fruitful one, providing a host of distinct observational results.

In *Lattice Gauge Theory* the space-time continuum is replaced by a point lattice, embedded into an Euclidean continuum, and on this lattice a discrete Euclidean version of QCD is formulated. The functional integrals on the basic matter fields (quarks) and gauge fields (gluons) reduce there to multiple Grassmann and Riemann integrals over a finite domain. The formidable object of this approach is to deduce confinement and the existence and properties of the observed hadrons. This approach is not restricted to weak coupling. It is still far from being a complete realization of a relativistic quantum field theory, since there is no proof of the existence of a continuum limit which in addition would have to allow a proper continuation to Minkowski space-time. It should be noted, however, that numerical results are promising. Comparing the two facets *Renormalized Perturbation Theory* and *Lattice Gauge Theory* which aim at concrete empirically adequate constructions, they point to complementary physical domains, namely to short and long distance effects.

This short summary should make it clear, that the complex *Relativistic Quantum Field Theory* is not a monolithic theory, but shows complementary facets which are united rather by a common ideal goal then by a general theory from which they could be derived.

0.7.2 The Standard Model as Symbolic Construction

In our perspective on relativistic quantum field theory, which is directed towards its constituting general aspects, we do not consider the multitude of models which combine elements of the facets expounded, augmented by further particular assumptions or approximations. Especially in QCD such models play a prominent role and in some cases they are even a prerequisite for a systematic treatment, as for instance the parton model for the application of perturbation theory in strong interactions.

We now come back to the intricate relation between particles and fields already alluded to in sect. 0.2.3. Making use of the material presented in sect. 0.3 we can expound our point of view, namely that constructions of increasing complexity have to be made in order to achieve a consistent theoretical description of the phenomena.

Especially the conversion of a classical field theory with local gauge symmetry into a quantum field theory requires novel constructions. Because of the *local* symmetry present in the classical Lagrangian a whole class of field configurations equivalent under gauge transformations, determines a physical configuration, in place of a single field configutation, see (24),(42). In the process of quantization within perturbation theory a unique representative of each class is selected by imposing an appropriate gauge condition. The Coulomb gauge fixing ($\vec{\partial} \cdot \vec{A} = 0$) provides such a selection, but it violates relativistic invariance in the construction, in addition a non-local interaction is introduced. In order to construct a systematic renormalized perturbation expansion the choice of a *covariant* gauge fixing condition appears to be indispensable; this in turn rules out a state space with a positive definite metric, (Strocchi (1970)). *"Unphysical states"* with partly negative norm have to be introduced (longitudinal and timelike photons), the corresponding field components, however, are treated in the course of the evaluation of Feynman diagrams exactly as the fields corresponding to the physical states.

In the case of non-Abelian gauge symmetry the construction has even to be extended. Because of the inherent self-interaction of the gauge fields, gauge fixing generates a non-local contribution, the Faddeev-Popov determinant, as an additional dynamical agent. This determinant can be implemented into the Lagrangian by introducing additional scalar local fields, ghost fields, see (35), which have to anticommute, thus violating the connection between spin and statistics. As a result of this construction a local Lagrangian, (36), is obtained which contains fully dynamical fields and has a non-degenerate quadratic part, forming the point of departure for developing a renormalized perturbation expansion. Moreover, this Lagrangian satisfies the BRST symmetry, which involves all fields entering and acts as local gauge symmetry on the non-ghost fields. For the inductive construction and the proof of renormalizability relativistic covariance and locality appear essential; there are non-covariant gauges, like the temporal gauge $A_0 = 0$, which would dispense from introducing ghost fields.

The full state space of the covariantly constructed quantum gauge field theory has an indefinite metric - thus not conform with Wightman postulate W1, see sect. 0.3.1. It contains, however, a subspace selected by subsidiary conditions, as indicated in sect. 0.4.2. The physical states have positive norm and the related physical S-matrix is unitary. The dynamical evolution encoded in this S-matrix, however, involves *all* fields entering the theory.

The unphysical degrees of freedom are essential parts of the mathematical structure of renormalized perturbative gauge field theory. In the facets of quantum field theory which concentrate on the foundational basis, local gauge symmetry is not dealt with and therefore the problem of unphysical states does not arise. In lattice gauge theory ghost fields do not occur. The integration over the gauge fields, indicated in (18), extends over a compact domain, the parameter space of the unitary matrices $u(x_i, x_j)$ of the gauge group, see sect. 0.4.3. Therefore the integration over *all* elements of the class representing the same physical configuration leads to a finite result and no selection of a particular representative is necessary. It is evident that this is a particular feature of the discretized theory because only there the integration over all representatives leads to a finite result.

We now discuss the implications of these specific constructions for the standard model of particle physics. A short glance on certain aspects of the history of weak interactions shows the constructive character of theory development and how the unity of the facets was an ideal goal, but not necessarily realized *de facto*. Even in an early state of quantum field theory the special problems connected with the ultraviolett divergences of a four-fermion interaction were recognized. This problem together with the other infinities encountered in the first attempts of perturbation theory led to the widespread belief that at high energies local quantum field theory breaks down and e.g. a fundamental length has to be introduced (see e.g. Heisenberg (1943)). After the establishment of renormalized perturbation theory in QED the construction of such a theory for weak interactions was a prominent goal. It was clear that the range of interaction for weak decays is very short, therefore an intermediate field mediating weak interaction had to be very massive. A massive vector field without additional symmetry requirements leads however to a non-renormalizable theory. Nevertheless many theories were constructed based on the exchange of massive vector fields and efforts were made to find the corresponding "intermediate bosons" experimentally. After the invention of the Higgs-Kibble mechanism, see sect. 0.4.4, Weinberg (1967) proposed in 1967 the mass generation of the intermediate boson by spontaneous breaking of a gauge symmetry. Based on this mechanism he could even make rough predictions for the masses of the gauge bosons, they were more than an order of magnitude larger than those of any particle known up to then. Weinberg acknowledged that there was no proof for the renormalizability of the theory, it was t'Hooft and Veltmann ('t Hooft (1971b); 1971a); 't Hooft and Veltman (1972b), see also the article of Veltman in (Hoddeson and others (1997)), p. 145ff. who undertook the formidable task of proving renormalizability of gauge theories with spontaneous symmetry breaking. This was an important achievement in the facet of renormalized perturbation theory, independently of its phenomenological applications. The positive theoretical result had also practical consequences: it was an additional encouragement, see the article of Perkins in (Hoddeson and others (1997)), p. 432 for the experimentalists who undertook the formidable task of finding the very rare events induced by neutral currents, characteristic for the theory. After the positive results one had enough information to predict the masses of the intermediate bosons with an accuracy of few percent and to establish them in a dedicated experiment (Arnison and others (1983a); 1983b)).

The renormalizability of a theory involving massive vector fields is based on the Higgs mechanism within a renormalizable gauge field theory. We have discussed this mechanism of mass generation and the renormalization procedure for an Abelian gauge theory in sect. 0.4.4 and the application to the Standard Model in sect. 0.5.3. Here we summarize the features indicating the constructive character of the theory. We shortly direct our attention to the peculiar features of the $SU(2)$ gauge theory and concentrate on the part of the Lagrangian analogue to (41), the problems of mixing with the electromagnetic field and the coupling to fermions will be discussed later. As classical precursor theory acts a $SU(2)$ gauge theory with a doublet of complex scalar fields as matter fields, the latter one in addition subject to a specific self-interaction. The energy density of this locally gauge invariant model shows the degeneracy of a sphere S^3. Spontaneous symmetry breaking is generated by selecting arbitrarily a particular minimum by a corresponding constant shift of the scalar doublet analogous to (43). This operation keeps the *local* $SU(2)$ gauge symmetry but breaks the originally a fortiori present *global* $SU(2)$ symmetry, and most important, provides all three gauge fields with a mass. The renormalized perturbation expansion developed from this point of departure has a state space with an indefinite metric. This total state space corresponds to the three vector fields, a scalar field, three Goldstone scalar fields, and ghost fields, the latter due to covariant gauge fixing; all these fields are massive. The "physical" state space is again singled out analogously to the procedure indicated in sect. 0.4.2. The particle states appearing in the physical state space are three vector mesons with three polarizations each and one scalar meson, which are identified with the observed charged vector bosons $W^{\pm}$, the Z^0-meson (after mixing with the photon) and the Higgs meson, still to be established experimentally.

The incorporation of this mechanism into the $U(1) \times SU(2)$ gauge symmetry of electroweak interactions (see sect. 0.5.3) poses further tasks, (Sibold (2000)). The unbroken symmetry $U(1)$ has to be established jointly in the process of renormalization and the photon has to be kept massless in mixing with one of the gauge fields of $SU(2)$ to form the Z^0 meson. As mentioned, the specific coupling of the gauge fields to fermions according to (50),(51)leads to anomalies, which destroy renormalizability of the purely leptonic sector.

In the perturbative treatment of the Higgs mechanism the vacuum expectation value ρ, breaking the global symmetry, see (43), plays a decisive role. It is the main ingredient for the mass generation of the gauge bosons. This expansion of a classical Lagrangian at an arbitrarily chosen point of a degenerate minimum provides a new classical Lagrangian, which itself forms the point of departure for the construction of a perturbative quantum field theory. Gauge fixing breaking the local gauge symmetry then appears mandatory in this endeavour. In the lattice regularized version of a gauge theory gauge invariance can be strictly kept, and in a special model (Elitzur (1975)) and more general in (De Angelis *et al.* (1978)) it was shown that the expectation value of the scalar field vanishes and thus the parameter ρ is equal to zero. This situation clarifies the symbolic nature of the concepts: Even if in the future the continuum limit could be rigorously shown to exist and even if quantitative results in lattice calculations could be achieved which are comparable in precision and reliability to those of perturbation theory, this would certainly not invalidate renormalized perturbation theory. It shows however that the expectation value of the scalar field is a quantity which makes only sense in the theoretical frame it is used and where it plays an essential role.

The observation of more and more strongly interacting particles, especially in the sense of definition 1 in sect. 0.2.1, led very early to the idea, that some of them correspond to fundamental fields and some are composite states of those. As early as 1949 Fermi and Yang (1949) discussed the possibility that for instance the mesons are bound states of nucleons and their antiparticles which were at that time not yet detected. But soon the opinion that quantum field theory was of little use in strong interactions (see e.g. Chew (1961)) was widely accepted and the search for fundamental fields seemed obsolete.

Today the theory of strong interactions, QCD, is based on fundamental fields; it is a renormalizable gauge theory of quark fields. But whereas the search for consistency was the driving force in shaping of the theory of electroweak interactions, the construction of QCD was largely driven by the goal to deal with phenomenological peculiarities. Objects which turned out later to give rise to the fundamental matter fields, the quarks, were introduced independently with different motivations. Gell-Mann (1964), using the method of current algebra found that many properties of hadrons, especially concerning their weak and electromagnetic interaction could heuristically be explained by assuming the existence of fundamental fields with peculiar quantum numbers (e.g. fractional charge and baryon number). Already in his original paper, however, he suggested that no asymptotic particles correspond to these fields and he called the quarks "mathematical particles" (see also Gell-Mann's comments in (Hoddeson and others (1997)), p. 625ff.) Independently, Zweig (1964) was investigating regularities and seeming paradoxes in strong interactions and came to the conclusion that they could be explained by the assumption that hadrons consist of fundamental particles, with the same peculiar quantum numbers as the quarks of Gell-Mann. The plausibility especially of Zweig's results led to an intensive search for these constituting particles. The outcome was negative and the present experimental and theoretical situation favours indeed strongly the original suggestion of Gell-Mann that no observable asymptotic states correpond to quark fields. For quite some time "quark models" in strong interactions were in a very heuristic state and relied on a more or less adventurous mixture of elements of perturbative quantum field theory, quantum mechanics and simplifying assumptions, an "effective field theory" in the widest sense.

As mentioned above the electroweak theory was in a theoretically much better state. Even before the proof of renormalizability of its perturbation expansion in 1972, there existed with Weinberg's Lagrangian at least the basis for a well defined theory which *could* be renormalizable and could serve as a basis for the inclusion of strongly interacting fields. It seemed therefore plausible to include also quark fields into the electroweak gauge theory in order to describe electroweak interactions of hadrons, though there was at that time no reliable field theoretical scheme to treat strongly interacting quark fields There was, however, a serious obstacle to the incorporation of quarks into the electroweak gauge theory, namely the absence (or extreme smallness) of flavour-changing weak neutral currents. By introducing a quark field with a new quantum number (called *charm*) as the weak isospin partner of the strange quark, Glashow, Iliopoulos, and Maiani (Glashow *et al.* (1970)) managed to suppress these contributions. After the detection of hadrons with this quantum number it was clear that these "charmed" quark fields had also to be taken into account in a theory of strong interactions.

The leptonic theory augmented by corresponding terms with quark fields opened not only the possibility to explain some weak decays of hadrons at least qualitatively, but it was also essential for the internal consistency, since it is free of anomalies! Hence quark fields played an essential role in the renormalized perturbation theory of electroweak interactions, that is in a consistent facet of quantum field theory. But the inclusion of quark fields made it necessary to extend the state space of leptons, to which asymptotic particles correspond and which therefore justly can be called physical space by fictious "physical states" of quarks, which have not been observed as asymptotic particles. On top of the real and fictitious physical states there occur in the full state space of perturbation theory also the unphysical states necessary for covariant gauge fixing.

When Gell-Mann, Fritzsch and Leutwyler (Gell-Mann (1972); Fritzsch *et al.* (1973)) proposed QCD, i.e. the non-Abelian $SU(3)$ gauge theory with quarks as matter fields as the fundamental theory of strong ineractions, emphasis was more on certain phenomenological aspects than on consistency. But the peculiar property of asymptotic freedom (see sect. 0.5.2) opened the way to use renormalized perturbation theory also in strong interactions. The corresponding state space is spanned by the "fictitious physical" states, quarks and transverse gluons and the "unphysical" states, namely longitudinal and timelike gluons and ghosts, all occuring in the propagators.

One of the most characteristic features of strong interaction, the rich hadron spectrum, is inherently unaccessible to a treatment by perturbation theory. For heavy quarks one can however take recourse to the consistent formalism of quantum mechanics. Here, after introducing a phenomenological potential, the hadronic states emerge consistently as bound states of two heavy quarks, perturbative QCD corrections can be included. In the Hilbert space of quantum mechanics only those states which are colour singlets are observable.

Lattice gauge theory is much closer to relativistic quantum field theory theory. Here, as mentioned in sect. 0.5, numerical results have been obtained for the lowest lying hadrons with given quantum numbers. To obtain higher excited states is difficult, since the extraction of the particle content from a numerically given Schwinger function is delicate. Stable particles correspond to poles in the Wightman functions and therefore in the Schwinger functions to correlation lengths (see sect. 0.4.3). In order to select a smaller correlation length (higher mass) one has to subtract the terms with the bigger ones. Furthermore it is unclear how an unstable particle, corresponding to a pole in the *complex* plane, can be treated consistently in the Euclidean theory. In order to get some information on decays one has to proceed analogously to perturbation theory, namely to calculate the transition probabilities and then calculate the width semiclassically. Numerical lattice calculations have also been performed to test certain aspects of effective theories which try to explain confinement, most notably the t'Hooft-Mandelstam model (Mandelstam (1976); 't Hooft (1978)) which is based on vortex lines in a condensate of magnetic monopoles. Another important domain of numerical lattice gauge calculations is the investigation of quantum field theories at finite temperatures, notably the transition to phases without confinement and without chiral symmetry breaking.

In the course of the last decades growing importance has been attached to the concept of effective quantum field theories (sect. 0.6), based on effective Lagrangians[29]. An early example of a Lagrangian, which now would be called effective, is the Euler-Heisenberg Lagrangian (Euler (1936)), and also Fermi's theory of four-fermion interaction can now be considered as an effective Lagrangian for the weak sector of the standard model; both lead to non-renormalizable theory. After the appearance and the phenomenological success of renormalized perturbation theory Lagrangians which did not lead to a renormalizable theory came in disrepute. When Weinberg proposed in 1968 such a "phenomenological" Lagrangian (Weinberg (1968)) he took care to state that it was only an auxiliary device to obtain easily results for pion scattering and production, which were in accordance with those obtained by a theory considered to be more fundamental, namely current algebra. Because of the lack of renormalizability, higher order corrections were considered to be meaningless.

The attitude changed, according to Weinberg (2009), in the second half of the seventies. He was influenced by Wilson, who investigating critical phenomena in statistical physics had introduced a variable ultraviolett cutoff, but had changed the "bare couplings" in the Lagrangian as to keep the physical quantities strictly cutoff independent. But even if the underlying theory is renormalizable, one also has to introduce additional "insertions" which are marginal, see sect. 0.6.1. Taken as interactions these terms would make the theory non-renormalizable. This formal development somewhat changed the view on the renormalizable theories. Since the occurence of fields with a high mass[30] outside the reach of present experiments can never be excluded, it is always possible that a renormalizable theory is only an

[29]These effective Lagrangians have to be distinguished from the effective action obtained by formal functional transformations of the original action

[30]massive fields are fields with a renormalized mass, they may or may not correspond to asymptotic particles

effective theory of a more extensive one with additional very heavy fields. An experimental hint to that would be the existence of processes which are forbidden in the effective theory through its genuine interaction terms, but possible through the insertions necessitated by the high masses. Proton decay, a much searched process, is an example of such a process. But also the attitude to non-renormalizable theories changed. The occurrence of formally non-renormalizable terms opened the way to consider Lagrangians with non-renormalizable terms as the basis of a dynamical theory, that is also to calculate higher orders with closed loops, provided more and more terms are introduced in order to ensure the independence of the observable results from the regulator, see sect. 0.6.2.

It is evident that from our point of view, from which theories are seen as symbolic constructions to explain phenomena, and not as faithful representations of an underlying reality, each theory can be called effective. We nevertheless admit that there are certain hierarchies in effective theories. Even if the chiral symmetry breaking through the QCD Lagrangian is by no means proven, QCD is certainly the more embracing theory and in lattice gauge theory one has started to calculate numerically the constants occurring in chiral perturbation theory. On the other hand one cannot obtain information on the small quark masses by renormalized perturbation theory of QCD, since the masses are much smaller than the scale where perturbative QCD can be applied. Therefore the information on the small quark masses is obtained by a mixture of chiral perturbation theory, perturbation theory and nonperturbative approaches. The large quark masses however can be identified through perturbation theory, where they give an important contribution due to their presence in the propagators.

0.7.3 The Symbolic Character of Quantum Field Theory Seen in a Historical Context

At several instances we have put forward arguments to view the achievements of quantum field theory in an epistemological perspective as symbolic constructions. In such a construction, based on a concept, the symbols entering the mathematical structure are not related directly to the phenomena, but only the relations obtained from this structure. The ghost fields in renormalizable (covariant) perturbation theory are examples.

The electromagnetic field, seen autonomously, exempt from a material carrier, was perhaps the first example of such a symbolic construction. It is thus no accident that Heinrich Hertz is one of the most outspoken and clearest protagonists of a symbolic view of natural sciences. Hertz made not only a great contribution to electromagnetism by discovering and detecting radio waves, but also was one of the first to emphasize the autonomous character of Maxwell's equations[31]. In insisting on the primacy of the equations he had freed the theory of Maxwell of ether mechanics, which with hindsight might be regarded as "ontological ballast".

In the introduction to his *Mechanics* Hertz makes some very concrete and lucid remarks on the task and the possibility of natural science. He considers as principal aim of conscious natural science (bewusster Naturerkenntnis) to foresee future experiences. In order to reach that he proposes a 'sign theory' for which he gives a set of rules, both formative and descriptive ,(Hertz (1894), pp. 1f).

> We form for us inner simulacra or symbols of the things and in such a way that the logical consequences of the symbols are always pictures of the physically necessary consequences of the depicted objects[32].

He does not take it for granted that such a procedure is possible, but notes that experience tells us, that it can be achieved. Hertz emphasizes the symbolic character of the theoretical entities (Hertz (1894), p. 2.):

> The images we speak of are our imaginations of the things; they have with the things the one essential correpondence which lies in the fulfillment of the above mentioned postulates.[33]

He also points out that by the congruence requirement the construction is by no means unique, but that different pictures can be distinguished according to their 'admissibility' (*Zulässigkeit*), 'correctness' (*Richtigkeit*) and

[31]The importance of Hertz in the development of the electromagnetic theory becomes evident by the fact that Einstein e.g. referred to Maxwell's theory as Maxwell-Hertz theory.

[32]'Wir machen uns innere Scheinbilder oder Symbole der äusseren Gegenstände, und zwar machen wir sie von folgender Art, dass die denknotwendigen Folgen der Bilder stets wieder Bilder seien von den naturnotwendigen Folgen der abgebildeten Gegenstände.'

[33]'Die Bilder, von welchen wir reden, sind unsere Vorstellungen von den Dingen; sie haben mit den Dingen die eine wesentliche Übereinstimmung, welche in der Erfüllung der genannten Forderung liegt.'

‘appropriateness’ (*Zweckmäßigkeit*). Pictures are admissible if they do not contradict logic (laws of thought), and admissible pictures are correct if ‘their essential relations do not contradict the relations between external objects’. However, admissible and correct pictures can (and normally will) differ in their appropriateness which comes in two guises; namely as ‘distinctness’ (*Deutlichkeit*) and ‘simplicity’ (*Einfachheit*). A picture which mirrors more of an object’s essential relations is called a clearer one, and amongst equally clear pictures the one with the less vacuous relations (i.e. relations which occur in the picture but not in the external world) is the simpler one. Such competing pictures normally occur if one is simplifying certain aspects of a more complete theory in models. Also, if one could obtain a direct dynamical development of local observables without recurse to local fields, this would be a simpler construction.

It is remarkable that already before the rise of quantum physics the philosopher Ernst Cassirer saw the forthcoming change in the epistemological attitude, namely away from science as mainly concerned with substances and their properties as fundamental objects towards the investigation of relations. This is clearly elaborated in his work *Substance and Function* (Cassirer (1910)). In his *Philosophy of Symbolic Forms*, where he expounds symbolic representation in all forms of human endeavour to grasp the world, he notes that indeed natural science was first to be aware of its symbolic character and he quotes intensively Hertz and Helmholtz (Cassirer (1923 1929), vol. I, p. 5 f, vol III, p. 522 ff).

The epistemologigal position to view a physical theory as a symbolic construction is distinctly opposed to both *positivism*, which advocates a mere descriptive role of the theory, and *entity realism*, according to which the mathematical entities of the theory express ultimate elements in nature. The concentration on Maxwell’s equation and discarding ether mechanics could at first sight be viewed as a typical move to positivism. Indeed, the younger Planck, who was an adherent of Ernst Mach, expresses in 1899 (Planck (1958), 1, p. 604) the view that the success of Maxwell’s theory is also a triumph of Mach’s positivism. But reducing the formalism to a kind of shorthand notation for tables connecting experimental data, as extreme positivism does, would not do justice to the wealth of relations generated by the involved mathematical quality of these equations. Hertz recognized this power very clearly. He writes admiringly on Maxwell’s equations : “One cannot read this beautiful theory without sometimes feeling as if those mathematical formulæ had their own life and intelligence, as if those were more clever than we, even more clever than their inventor, as if they would yield more than was put into them at the time" (Hertz (1889), p. 11).

From the mathematical form of a theory may follow startling predictions of novel phenomena. This applies particularly to quantum field theory. Perhaps the most spectacular triumph was the prediction of anti-matter by Dirac[34]. Other examples are the prediction of the charm quark field and the mass of the top quark. Those who advocate entity realism, take the successful predictions of certain theories as a strong argument for their view. One of the strongest arguments against this realism is the pessimistic meta-induction: In the past all theories have turned out to be wrong if applied generally, i.e. not restricted to a limited domain, so present theories most probably will suffer the same fate.

We shall not go here in a general discussion, but concentrate on quantum field theory. First we shall in some detail show that quantum field theory, viewed as a faithful image of the "real world", leads to serious problems. From a realistic point of view one could argue that the infinities occurring in a naive application of perturbation theory are only due to a extrapolation of an effective theory into domains where it is not applicable and where new effects do occur. Therefore on should deduce the terms in the perturbative series in a direct way, but limit the integration range of the multiple momentum space integrals (see sect. 0.3.4) by a large, but fixed and finite cutoff. In this way all terms of the (formal) power series are well defined. By choosing this fixed cutoff large enough and by adjusting the constants in the classical Lagrangian, i.e. the bare constants, one obtains expressions which are arbitrarily close to the observations as are the results of renormalized perturbation theory.

But an important point in favour of the interpretation of quantum field theory as a symbolic construction rather than a faithful representation of reality is the problem of the cosmological constant, see e.g. (Straumann (1999); Trodden and Carroll (2004)). The vacuum expectation value of the contracted energy momentum tensor is proportional to the energy density of the vacuum, $\rho_{\rm vac} = \frac{1}{4}(\Omega, T_{\mu\nu}\Omega)g^{\mu\nu}$ and hence contributes to the Einstein equations of gravity as a contribution to the cosmological constant. From astronomical data one deduces a limit on the energy density of the universe $\rho_{\rm vac} \leq 10^{-46}$ GeV^4. In a naive treatment of perturbative quantum field theory

[34]The first hint was given by relativistic quantum mechanics, but a consistent description could only be given in quantum field theory, and most notably through the CPT theorem

calculation of the energy momentum tensor leads to one of the bewildering infinities, even in free field theory. It is to be avoided in a renormalized theory. Since in relativistic quantum field theory excluding gravity, only energy differences are observable, one can choose the *renormalized* vacuum energy density to be zero or any finite number. From a realistic point of view, as the one described above, one has however no such freedom, the energy density of the vacuum can be calculated and is proportional to the 4th power of the cutoff. Since the validity of the standard model, as it stands, extends certainly far into the GeV region, the cutoff has to be at least of this order of magnitude. The thus obtained cosmological constant is at least 50 orders of magnitude larger than the phenomenological one[35]. Adopting a realistic view one has to postulate an up to now unknown mechanism which compensates the vacuum energy from quantum field theory to an accuracy of more than 50 orders of magnitude!

We have stressed several times that the notion of a "particle" is not independent of the theoretical frame. This holds to some extend even for the electron, which seems a well established stable particle. In no experiment it does show up as an isolated state, but always together with (in principle an unlimited number of) soft photons.

There is however an apparent degree of concreteness of the categories of "particles". It can be easily accounted for in the frame of symbolic constructions (theories). The electron can – and for certain purposes even must – be considered as a charged classical particle, subject to the laws of mechanics, subject to force fields given by classical electrodynamics. It also appears in the well defined and closed theory of non-relativistic quantum mechanics, as a fundamental constituent of matter. Remarkably the numerical values for mass and charge are the same there as in the frame of classical mechanics. Heavy quarks can also be incorporated into the frame of non-relativistic quantum mechanics or more ambitiously of non-relativistic QCD. In a stringent theoretical frame light quarks and gluons have to be considered as fundamental fields in relativistic quantum field theory, both in its perturbative and nonperturbative facet. In lattice gauge theory they are basic fields generating hadrons dynamically. In perturbation theory the quark and gluon fields are directly related to physical particle states. The values

[35]In supersymmetric theories the cosmological constant is zero to any order of perturbation theory, since the fermionic contributions are cancelled by bosonic ones. Since supersymmetry is not observed, it is, if it prevails, certainly broken in the TeV range and therefore does not cure the problem

of the masses and charges in quantum field theory are on a quite different footing than in classical physics or quantum mechanics, since they have to be fixed via the choice of renormalization conditions. For those particles which also occur as asymptotic particles the renormalized quantities can be related to the classical quantities, but for quarks, e.g., the mass is only an internal parameter, occurring e.g. in the field propagator. The ghost fields finally only appear in (covariant) renormalized perturbation theory but they do not correspond to states in the physical subspace. Whereas the notation "particle" is often used loosely, the statement that a certain field is a necessary element of a given symbolic construction is unambiguous.

The view of a symbolic construction leaves aside attempts to attribute to the mathematical symbolism an ontological status. Instead, its criterion is the usefulness of the concept inherent in the construction in accounting for a significant empirical domain - in short, as clearly formulated by Hertz, to make correct predictions. This moderate position acknowledges in the large the coexistence of autonomous closed physical theories, like classical mechanics, electrodynamics, quantum mechanics,..., accounting for a respective large class of phenomena. Moreover, this position meets the possibility - actually the norm, seen historically - that the full mathematical range of a theory is (much) larger than its physical domain of validity.

Already before Hertz, Helmholtz in his considerations on the role of signs, still related to sensual impressions recognized: the most important feature of the signs is that they map the *law* of what is happening. Helmholtz and Hertz could speak of "*the* laws", since they could not yet know that the application of both Newtonian mechanics and classical electrodynamics have only a limited domain of validity even in their proper fields of application. Henri Poincaré however saw the dawn of unlimited applicability of Newtonian mechanics, but nevertheless he stresses the importance of the formalism on the expense of the underlying signals. Famous is the account of the validity of Fresnel's formulae of inflection and refraction (Poincaré (1927), X, p. 190.)

> [...] the aim of Fresnel was not to find out whether there really is an ether, whether it is or it is not formed of atoms, whether these atoms really move in this or that sense; ... these appellations were only images substituted for the real objects which nature will eternally hide from us. The true relations between these real objects are the only reality we can attain to, and the only condition is that the same relations exist between these objects

> as between the images by which we are forced to replace them. If these relations are known to us, what matter if we deem it convenient to replace one image by another.[36]

The constructive character becomes even clearer in the following passage near the end of *The Value of Science* (Poincaré (1905), XI, p. 296.):

> In summary, the only objective reality are the relations between the things from which the universal harmony starts. Doubtless these relations, this harmony, would not be conceived outside a mind who conceives or feels them. But they are nevertheless objective since they are, become, or stay common to all thinking beings.[37]

This (relative) stability of the relations has given rise to structural realism, which by Cao and Schweber (1993)[38] has been advocated as the adequate epistemological position with regard to quantum field theory. This position is certainly much closer than positivism and entity realism to the view of symbolic constructions. We nevertheless consider the view of symbolic construction more adequate, as we have elaborated in some detail in (Dosch *et al.* (2005)). There special emphasis was given to a semiotic interpretation, which accepts the different facets of quantum field theory as different codes.

[36]'[...] le but de Fresnel n'était pas de savoir s'il y a réellement un éther, s'il est ou non formeé d'atomes, si ces atomes se meuvent réellement dans tel ou tel sense; c'était de prévoir les phénomènes optiques. Or, cela, la théorie de Fresnel le permet toujours, aujourd'hui aussi bien qu'avant Maxwell. Les équations differentielles sont toujours vraies; [...] Elles nous apprennent, après comme avant, qu'il y a tel rapport entre quelque chose et quelque autre chose; seulement, ce quelque chose nous l'appelions autrefois *mouvement*, nous l'appelons maintenant *courant électrique*. Mais ces appelations n'étaient que des images substituées aux objets réels que la nature nous cachera éternellement. Les rapports véritables entre ces objets réels sont la seule réalité que nous puissions atteindre, et la seule condition, c'est qu'il y ait les mêmes rapports entre ces objets qu'entre les images que nous sommes forcés de mettre à leur place. Si ces rapports nous sont connus, qu'importe si nous jugeons commode de remplacer une image par une autre.'

[37]'En résumé, la seule réalité objective, ce sont les rapports des choses d'où résulte l'harmomie universelle. Sans doutes ces rapports, cette harmonie ne sauraient être conçus en dehors d'un esprit qui les conçoit ou qui les sent. Mais ils sont néanmois objectifs parce qu'ils sont, deviendront, ou resteront communs à tous les êtres pensants.'

[38]Epistemological considerations concerning quantum field theory are the subject of several books and conferences, e.g. Auyang (1995); Cao (1996); 1999)

In the Standard Model of particle physics the concept of gauge symmetry has reached its climax; it seems appropriate to conclude with a quotation of the founder of this concept, Hermann Weyl (Weyl (1949))[39]:

> [...] it is the free in symbols acting spirit which constructs himself in physics a frame to which he refers the manifold of phenomena. He does not need for that imported means like space and time and particles of substance; he takes everything from himself[40]

Acknowledgements. The authors gratefully acknowledge valuable suggestions made by the referees. One of us (HGD) wants to thank John Stachel for interesting discussions and suggestions.

[39] The symbolic constructivism of Weyl is a principal subject of Sieroka (2010)

[40] '[...] dass es der freie, in Symbolen schaffende Geist ist, der sich in der Physik ein objektives Gerüst baut, auf dass er die Mannigfaltigkeit der Phänomene ordnend bezieht. Er bedarf dazu keiner solchen von aussen gelieferten Mittel wie Raum, Zeit und Substanzpartikel; er nimmt alles aus sich selbst'.

References

G. Altarelli. Partons in quantum chromodynamics. *Phys. Rep.*, 81:1–129, 1982.

H.L. Anderson, E. Fermi, E.A. Long, R. Martin, and D.E. Nagle. Total cross section of negative pions in hydrogen. *Phys. Rev.*, 85:934–935, 1952.

C.D. Anderson. The positive electron. *Phys. Rev.*, 43:491–494, 1933.

T. Appelquist and J. Carrazone. Infrared singularities and massive fields. *Phys. Rev.*, D11:2856–2861, 1975.

G. Arnison et al. Experimental observation of isolated large transverse energy electrons with associated missing energy at $s^{1/2} = 540$ GeV. *Phys. Lett.*, B122:103–116, 1983.

G. Arnison et al. Experimental observation of lepton pairs of invariant mass around 95-GeV/c^2 at the CERN SPS collider. *Phys. Lett.*, B126:398–410, 1983.

A. Aste, M. Dütsch, and G. Scharf. Perturbative gauge invariance: the electroweak theory II. *Annalen Phys.(Leipzig)*, 8:389–404, 1999.

S.Y. Auyang. *How Is Quantum Field Theory Possible*. Oxford University Press, Oxford, 1995.

T. Balaban. Large field renormalization.I. The basic step of the R operation. *Commun. Math. Phys.*, 122:175–202, 1989.

T. Balaban. Large field renormalization.II. Localization, exponentiation and bounds for the R operation. *Commun. Math. Phys.*, 122:355–392, 1989.

C. Becchi, A. Rouet, and R. Stora. Renormalization of gauge theories. *Ann. Phys.*, 98:287–321, 1976.

F. Bloch and A. Nordsieck. Note on the radiation field of the electron. *Phys. Rev.*, 52:54–59, 1937.

S.A. Bludman. On the universal Fermi interaction. *Nuov. Cim.*, 9:433–445, 1958.

N.N. Bogoliubov and O.S. Parasiuk. Über die Multiplikation der Kausalfunktionen in der Quantentheorie der Felder. *Acta Math.*, 97:227–266, 1957.

N.N. Bogoliubov and D.V. Shirkov. *Introduction to the Theory of Quantized Fields*. Wiley-Interscience, New York, 1 edition, 1959.

C. Bouchiat, J. , Iliopoulos, , and Ph. Meyer. An anomaly-free version of Weinberg's model. *Phys. Lett.*, 38 B:519–523, 1972.

R. Brunetti and K. Fredenhagen. Microlocal analysis and interacting quantum field theories: Renormalization on physical backgrounds. *Commun. Math. Phys.*, 208:623–661, 2000.

R. Brunetti and K. Fredenhagen. Quantum field theory on curved backgrounds. In C. Bär and K. Fredenhagen, editors, *Quantum Field Theory on Curved Spacetimes*. Springer, 2009.

D. Buchholz and R. Haag. The quest for understanding in relativistic quantum physics. *J. Math. Phys.*, 41:3674–3697, 2000.

D. Buchholz, M. Porrmann, and U. Stein. Dirac versus Wigner. towards a universal particle concept in local quantum field theory. *Phys.Lett B*, 267:377–381, 1991.

T.Y. Cao and S.S. Schweber. The Conceptual Foundations and the Philosophical Aspects of Renormalization Theory. *Synthese*, 97(1):33–108, 1993.

T.Y. Cao. *Conceptual developments of 20th century field theories*. Cambridge University Press, Cambridge, UK, 1996.

T.Y. Cao, editor. *Conceptual foundations of quantum field theory*. Cambridge University Press, 1999.

E. Cassirer. *Substanzbegriff und Funktionsbegriff*. Bruno Cassirer, Berlin, 1910. Authorized English translation, Dover 1953, Classic reprint 2010.

E. Cassirer. *Philosophie der Symbolischen Formen*. Bruno Cassirer, Berlin, 1923-1929.

F.G. Chew. *S-Matrix Theory of Strong Interactions*. Benjamin, New York, 1961.

M. Creutz. *Quarks, Gluons and Lattices*. Cambridge University Press, Cambridge, UK, 1985.

G.F. De Angelis, D. de Falco, and F. Guerra. Note on the abelian Higgs-Kibble model on a lattice: Absence of spontaneous magnetization. *Phys. Rev.*, D17:1624–1628, 1978.

M. Disertori and V. Rivasseau. Continous constructive fermionic renormalization. *Ann. Henri Poincaré*, 1:1–57, 2000.

H.G. Dosch and V.F. Müller. Renormalization of quantum electrodynamics in an arbitrary strong time independent external field. *Fortschr. Phys.*, 23:661–689, 1975.

H.G. Dosch, V.F. Müller, and N. Sieroka. *Quantum Field Theory in a Semiotic Perspective*. Springer, Berlin, 2005.

H.P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder, and K. Yamazaki. Zur Theorie der Elementarteilchen. *Z. Naturforschung*, 14A:441–485, 1959.

M. Dütsch and G. Scharf. Perturbative gauge invariance: the electroweak theory. *Annalen Phys.(Leipzig)*, 8:359–387, 1999.

F.J. Dyson. The radiation theories of Tomonaga, Schwinger, and Feynman. *Phys. Rev.*, 75:486–502, 1949.

F.J. Dyson. The S matrix in quantum electrodynamics. *Phys. Rev.*, 75:1736–1755, 1949.

F. Elitzur. Impossibility of spontaneously breaking local gauge theories. *Phys. Rev.*, D12:3978–3982, 1975.

F. Englert and R. Brout. Broken symmetry and the mass of gauge vector mesons. *Phys.Rev.Lett.*, 13:321–323, 1964.

H. Epstein and V. Glaser. The role of locality in perturbation theory. *Ann. Inst. H. Poincaré*, 19:211–2955, 1973.

H. Euler. Über die Streuung von Licht an Licht nach der Diracschen Theorie. *Ann. d. Phys.*, 26:398, 1936.

L.D. Faddeev and A.A. Slavnov. *Gauge Fields: An Introduction to Quantum Theory*. Addison-Wesley, 2 edition, 1991.

J. Feldman, J. Magnen, V. Rivasseau, and R. Sénéor. A renormalizable field theory: the massive Gross-Neveu model in two dimensions. *Comm. Math. Phys.*, 103:67–103, 1986.

E. Fermi and C.N. Yang. Are mesons elementary particles? *Phys. Rev.*, 76:1739, 1949.

E. Fermi. An attempt of a theory of beta-radiation. *Z. Phys.*, 88:161–177, 1934.

R.P. Feynman. Quantum theory of gravitation. *Acta Phys. Polonica*, 26:697, 1963.

H. Fritzsch, M. Gell-Mann, and H. Leutwyler. Advantages of the color oktet gluon picture. *Phys. Lett.*, 47B:365–368, 1973.

J. Fröhlich, G. Morchio, and F. Strocchi. Charged sectors and scattering states in quantum electrodynamics. *Ann. Phys.*, 119:241–284, 1979.

K. Gawedzki and A. Kupiainen. Gross-Neveu model through convergent perturbation expansion. *Comm. Math. Phys.*, 102:1–30, 1985.

M. Gell-Mann and M. Levy. The axial vector current in beta decay. *Nuovo Cim.*, 16:705–726, 1960.

M. Gell-Mann. A schematic model of baryons and mesons. *Phys. Lett*, 8:214–215, 1964.

Gell-Mann. Quarks and what else? *Acta Physica Austriaca, Suppl.*, 9:733, 1972.

S.L. Glashow, J. Iliopoulos, and L. Maiani. Weak interactions with lepton-hadron symmetry. *Phys. Rev.*, D2:1285–1292, 1970.

J. Glimm and A. Jaffe. *Quantum Physics*. Springer, New York, 2 edition, 1987.

J. Goldstone, A. Salam, and S. Weinberg. Broken Symmetries. *Phys. Rev.*, 127:965–970, 1962.

J. Goldstone. Field Theories with Superconductor Solutions. *Nuovo Cim.*, 19:154–164, 1961.

D.J. Gross and F. Wilczek. Ultraviolet behavior of non-abelian gauge theories. *Phys. Rev. Lett.*, 30:1343–1346, 1973.

G.S. Guralnik, C.R. Hagen, and T.W. Kibble. Global conservation laws and massless particles. *Phys.Rev.Lett.*, 13:585–587, 1964.

R. Haag. *Local Quantum Physics: Fields, Particles, Algebras*. Springer, 2 edition, 1996.

M.Y. Han and Y. Nambu. Three-triplet model with double SU(3) symmetry. *Phys. Rev. 139*, B1006-1010, 1965.

W. Heisenberg. Die beobachtbaren Grössen in der Theorie der Elementarteilchen, I und II. *Z. Phys.*, 120:513, 673, 1943.

K. Hepp. *Théorie de la renormalisation.* Springer, Berlin, 1969.

Heinrich Hertz. Über die Beziehung von Licht und Elektricität. Strauss, Bonn, 1889. Also in Gesammelte Werke, Bd I.

Heinrich Hertz. *Die Prinzipien der Mechanik.* Barth, Leipzig, 1 edition, 1894. with a preface of H. Helmholz; English translation, Classic Reprint 2010.

P.W. Higgs. Broken symmetries, massless particles and gauge fields. *Phys.Lett.*, 12:132–133, 1964.

L. Hoddeson et al., editors. *The Rise of the Standard Model.* Cambridge University Press, 1997.

S. Hollands and R.M. Wald. Existence of local covariant time ordered products of quantum fields in curved spacetime. *Commun. Math. Phys.*, 231:309–345, 2002.

C. Itzykson and J.-B. Zuber. *Quantum Field Theory.* McGraw-Hill, New York, 1 edition, 1980.

J. D. Jackson and L. B. Okun. Historical roots of gauge invariance. *Rev. Mod. Phys.*, 73:663–680, 2001.

J.M. Jauch and F. Rohrlich. *The Theory of Photons and Electrons.* Springer, New York, 2 edition, 1980.

R. Jost. *The General Theory of Quantized Fields.* American Mathematical Society, Providence,RI, 1965.

D. Kastler, editor. *The Algebraic Theory of Superselection Sectors. Introduction and Recent Results.* World Scientific, 1990.

N Kemmer. Isospin. In *Journal de Physique, International Colloqium on the History of Particle Physics*, volume 43 of *C-8*, page 359, Les Ulis, 1982. les editions de Physique.

C. Kim. A renormalization group flow approach to decoupling and irrelevant operators. *Ann. Phys.*, 243:117–143, 1995.

O. Klein. On the Theory of Charged Fields. In *New Theories in Physics*, page 77. Int. Inst. of Intellectual Co-operation,Warsaw, 1938.

C.M.G. Lattes, H. Muirhead, G.P.S. Occhialini, and C.F. Powell. Processes involving charged mesons. *Nature*, 159:694–697, 1947.

H. Lehmann, Symanzik. K., and W. Zimmermann. On the formulation of quantized field theories. *Nuovo Cimento*, 1:205–225, 1955.

H. Leutwyler. On the foundations of chiral perturbation theory. *Ann. Phys.*, 235:165–203, 1994.

J.H. Lowenstein. Convergence theorems for renormalized Feynman integrals with zero-mass propagators. *Comm. Math. Phys.*, 47:53–68, 1976.

M. Lüscher. Computational strategies in lattice QCD. 2010. arXiv 1002.4232 (Les Houches Summer School 2009).

S. Mandelstam. Vortices and quark confinement in nonabelian gauge theories. *Phys. Rep.*, 23:245–249, 1976.

I. Montvay and G. Münster. *Quantum Fields on a Lattice*. Cambridge University Press, Cambridge, 1997.

G. Morchio and F. Strocchi. Infrared problem, Higgs phenomenon and long range interactions. In G. Velo and A.S. Wightman, editors, *Fundamental Problems of Gauge Field Theory*, New York, 1986. Plenum Press.

[Particle Data Group Collaboration] Nakurama, K. *et al.* Review Of Particle Physics. *J. Phys.*, G 37:075021, 2010. `http://pdg.lbl.gov`.

L. O'Raifeartaigh and N. Straumann. Gauge theory: Historical origins and some modern developments. *Rev. Mod. Phys.*, 72:1–23, 2000.

K. Osterwalder and R. Schrader. Axioms for Euclidean Green's functions.I. *Commun. Math. Phys.*, 31:83– 112, 1973.

K. Osterwalder and R. Schrader. Axioms for Euclidean Green's functions. II. *Commun. Math. Phys.*, 42:281– 305, 1975.

K. Osterwalder and E. Seiler. Gauge field theories on a lattice. *Ann. Phys.*, 110:440–471, 1978.

Max Planck. *Physikalische Abhandlungen und Vorträge*. Vieweg, Braunschweig, 1958.

Henri Poincaré. *La Valeur de la Science.* Flammarion, Paris, 1905.

Henri Poincaré. *La Science et l'Hypothèse.* Flammarion, Paris, 1927.

H.D. Politzer. Reliable perturbative results for strong interactions? *Phys. Rev. Lett.*, 30:1346–1349, 1973.

H.J. Rothe. *Lattice Gauge Theories, An Introduction.* World Scientific, Singapore, 2005.

G. Scharf. *Quantum Gauge Theory – a True Ghost Story.* Wiley, New York, 2001.

B. Schroer. Infrateilchen in der Quantenfeldtheorie . *Fortschr. Phys.*, 11:1–33, 1963.

S. Schweber. *QED and the Men who Made it.* Princeton University Press, Princeton, 1994.

J. Schwinger. A theory of the fundamental interaction. *Ann. Phys (NY)*, 2:407, 1957.

N. Seiberg and E. Witten. Electro-magnetic duality, monopole condensation, and confinement in N = 2 supersymmetric Yang-Mills theory. *Nucl. Phys.*, B426:19–52, 1994.

E. Seiler and I-O. Stamatescu, editors. *Approaches to Fundamental Physics.* Springer, 2007. Lecture Notes in Physics 721.

E. Seiler. *Gauge Theories as a Problem of Constructive Quantum Field Theory and Statistical Mechanics.* Springer, Berlin, 1982.

K. Sibold. Algebraic methods of renormalization. In H. Latal and W. Schweiger, editors, *Methods of Quantization.* Springer, 2000.

N. Sieroka. *Umgebungen. Symbolischer Konstruktivismus im Anschluss an Hermann Weyl und Fritz Medicus.* Chronos, Zürich, 2010.

O. Steinmann. *Perturbative Quantum Electrodynamics and Axiomatic Field Theory.* Springer, Berlin, 1 edition, 2000.

N. Straumann. Zum Ursprung der Eichtheorien bei Hermann Weyl. *Phys. Bl.*, 43:414–421, 1987.

N. Straumann. The mystery of the cosmic vacuum energy density and the accelerated expansion of the universe. *Eur. J. Phys.*, 20:419, 1999.

R.F. Streater and A.S. Wightman. *PCT, Spin and Statistics, and all That.* Benjamin, Reading, MA, 3 edition, 1980.

F. Strocchi. Gauge problems in quantum field theory. iii. *Phys. Rev.*, D2:2334–2340, 1970.

F. Strocchi. *Selected Topics on the General Properties of Quantum Field Theory.* World Scientific, 1993.

F. Strocchi. *Symmetry Breaking.* Springer, Berlin Heidelberg, 2 edition, 2008. Lecture Notes in Physics, 732.

G. 't Hooft and M. Veltman. Combinatorics of gauge fields. *Nucl. Phys.*, B50:318–353, 1972.

G. 't Hooft and M. Veltman. Regularization and renormalization of gauge fields. *Nucl Phys.*, B44:189–213, 1972.

G. 't Hooft. Renormalizable Lagrangians for massive Yang-Mills fields. *Nucl. Phys.*, B35:167–188, 1971.

G. 't Hooft. Renormalization of massless Yang-Mills fields. *Nucl. Phys.*, B33:173–199, 1971.

G. 't Hooft. On the phase transition towards permanent quark confinement. *Nucl. Phys.*, B138-25:1–25, 1978.

Y. Takahashi. On the generalized Ward identities. *Nuovo Cimento*, 6:371–375, 1957.

J.C. Taylor. Ward identities and charge renormalization of the Yang-Mills field. *Nucl. Phys.*, B33:436–444, 1971.

M. Trodden and S.M. Carroll. TASI lectures: Introduction to cosmology. 2004. arXiv astro-ph/0401547.

I.V. Tyutin. Gauge invariance in field theory and statistical mechanics. *Lebedev Institute, Report No: FIAN-39*, 1975.

R.M. Wald. The formulation of quantum field theory in curved spacetime. *arXiv 0907.0416*, 2009.

J.C. Ward. An identity in quantum electrodynamics. *Phys. Rev.*, 78:182, 1950.

F. Wegner. Duality in generalized Ising models and phase transitions without local order parametrs. *J. Math. Phys.*, 12:2259–2272, 1971.

S. Weinberg. A model of leptons. *Phys. Rev. Lett*, 19:1264, 1967.

S. Weinberg. Nonlinear realization of chiral symmetry. *Phys. Rev.*, 166:1568–1577, 1968.

S. Weinberg. Phenomenological Lagrangians. *Physica*, A96:327–340, 1979.

S. Weinberg. Effective field theory, past and future. *PosCD09:001*, 2009. arXiv:0908.1964v3 [hep-th].

H. Weyl. Gravitation und Elektrizität. *Sitz. Preus. Akad. Wiss.*, page 465, 1918.

H. Weyl. Reine Infinitesimalgeometrie. *Math. Z.*, 2:384, 1918.

H. Weyl. Eine neue Erweiterung der Relativitätstheorie. *Ann. d. Phys.*, 59:101–133, 1919.

Hermann Weyl. *Raum, Zeit, Materie.* Springer, Berlin, 1923. paperback edition: Space, Time, Matter.

H. Weyl. Elektron und Gravitation I. *Z. Phys.*, 56:330–352, 1929.

Hermann Weyl. Wissenschaft als symbolische Konstruktion des Menschen. *Eranos-Jahrbuch*, page 375, 1949. Reprinted in Gesammelte Abhandlungen, Bd. IV, K. Chandraskharan ed., Berlin 1968.

K. G. Wilson and W. Zimmermann. Operator product expansions and composite field operators in the general framework of quantum field theory. *Commun. Math. Phys.*, 24:87–106, 1972.

Kenneth G. Wilson. Nonlagrangian models of current algebra. *Phys. Rev.*, 179:1499–1512, 1969.

Kenneth G. Wilson. The Renormalization Group and Strong Interactions. *Phys. Rev.*, D3:1818–1846, 1971.

K.G. Wilson. Confinement of quarks. *Phys. Rev.*, D10:2445–2459, 1974.

C.N. Yang and R.L. Mills. Conservation of isotopic spin and isotopic gauge invariance. *Phys. Rev.*, 96:191–195, 1954.

W. Zimmermann. Convergence of Bogoliubov's method of renormalization in momentum space. *Commun. Math. Phys.*, 15:208–234, 1969.

W. Zimmermann. Local Operator Products and Renormalization in Quantum Field Theory. In *Lectures on Elementary Particles and Quantum Field Theory*, volume 1, Cambridge, MA, 1970. M.I.T Press.

W. Zimmermann. Composite operators in the perturbation theory of renormalizable interactions. *Ann. Phys.*, 77:536–569, 1973.

W. Zimmermann. Normal products and the short distance expansion in the perturbation theory of renormalizable interactions. *Ann. Phys.*, 77:570–601, 1973.

G. Zweig. An SU(3) Model for Strong interactions Symmetry and its Breaking. *Cern Preprint*, 8419/Th412, 1964.