Annotations to

Beyond the Nanoworld

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Chapter 1

The Heroic Time

In this appendix I use, unless otherwise stated, natural units, that is $\hbar = c = 1$ and energy includes the rest energy; the mass is always the invariant (rest) mass.

1.2 Brave Old World

In order to determine the ratio of the electric charge of an electron to its mass, e/m, one uses both the effect of the Coulomb force of the electric field E and of the Lorentz force of the magnetic field B: p. 7
The Coulomb force is : $K = eE$ in direction of the E field
the Lorentz force is : $K = e\frac{v}{c}B$, it is perpendicular to the direction of the velocity v of the charged particle and the magnetic field B.
Like always, c is the vacuum velocity of light.

•The Rutherford scattering formula for particles of mass m, of charge e and of velocity v ($v \ll c$) at a heavy nucleus of charge Ze is : p. 9f

$$\frac{d\sigma}{d\Omega} = \frac{Z^2 e^4}{4m^2 v^4 \sin^4(\theta/2)} \tag{1.1}$$

where θ is the scattering angle.

1.3 Detection of Particles

The energy loss by ionization per unit length of a particle with charge e, mass m and velocity v is given by the semiclassical formula: p. 16ff

$$\frac{dE}{dx} \approx Kz^2 \frac{Z}{A} \frac{c^2}{v^2} \left(\frac{1}{2} \log \frac{2m_e\, v^2\, T_{max}}{I^2(1 - v^2/c^2)} - \frac{v^2}{c^2} - \frac{\delta}{2} \right) \tag{1.2}$$

Here T_{max} is the maximum kinetic energy which can be imparted to a free electron in a single collision, z is the charge of the ionizing particle (in elementary units), v its velocity, I is the mean excitation energy, A is the molecular weight (g/mol) of the stopping material, $K = 0.307$ MeV cm^2 / g, m_e is the elctron mass and δ a material constant taking care of polarization effects.

For small velocities, ($v \ll c$), the ionization density is inversely proportional to the square of the velocity and thus allows to measure the latter. If one knows also the momentum of the particle, e.g. from the curvature of the trajectory in an magnetic field, on can determine the mass of the ionizing particle from momentum and velocity.

For large velocities ($v \approx c$) the energy dependence is weak.

The Mean energy loss rates for different materials are displayed in Figure 1.1

1.4.1 Special Relativity and Quantum Physics

The 4 vectors play in the 4 dimensional relativistic space-time continuum the same role as usual 3 vectors in three dimensional space. p. 22f

$$a = (a_0, a_1, a_2, a_3). \tag{1.3}$$

Space and time coordinates form together a 4 vector:

$$x = (x_0 = xt, x_1, x_2, x_3) \tag{1.4}$$

The scalar product of two 4 vectors is:

$$a.b = a_0 b_0 - a_1 b_1 - a_2 b_2 - a_3 b_3 \tag{1.5}$$

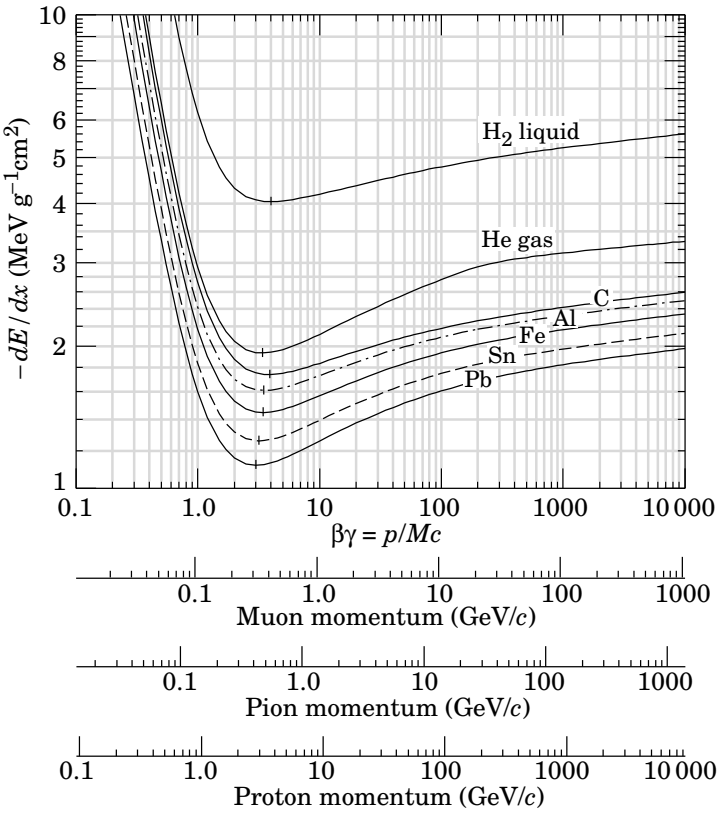

Figure 1.1: Mean energy loss by ionization in different materials, From Particle Data Group, Figure 27.3

It can be expressed with the *metric tensor* (4×4 matrix):

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1.6}$$

$$a.b = \sum_{\mu,\nu=0}^{3} a_\mu b_\nu g_{\mu\nu} \tag{1.7}$$

(Stricly speaking it is a pseudo-metric tensor, since it is not positive definite. A bit of matrix algebra will be explained below equation (1.28) ff)

Loke the rotations in three dimensional space, Lorentz transformations leave the 4 dimensional scalar product unchanged.

Energy E and momentum $\vec{p} = (p_1, p_2, p_3)$ form together a 4 vector:

$$p = (E/c = p_0, p_1, p_2, p_3), \tag{1.8}$$

The scalar product of p with itself is the product of mass and the square of the vacuum velocity of light:

$$p.p = E^2/c^2 - \vec{p}^2 = m^2c^2 \tag{1.9}$$

The relation between energy E, 3 momentum $\vec{p}$ and velocity $\vec{v}$ of a particle with mass m is therefore:

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}; \quad E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \tag{1.10}$$

• The free Dirac equation (that is without electromagnetic interaction) is: p. 24

$$\left(\gamma_0 \frac{\partial}{\partial x_0} + \gamma_1 \frac{\partial}{\partial x_1} + \gamma_2 \frac{\partial}{\partial x_2} + \gamma_3 \frac{\partial}{\partial x_3} - mc^2\right) u(\vec{x}, t) = 0 \tag{1.11}$$

where the γ matrices are 4×4 matrices with the following properties:

$$\gamma_\mu \cdot \gamma_\nu + \gamma_\nu \cdot \gamma_\mu = 2g_{\mu\nu}. \tag{1.12}$$

The γ matrices can be constructed from the 2×2 spinor matrices σ_k (see 1.40) and the 2×2 unit matrix $\mathbf{1}$:

$$\gamma_0 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \gamma_k = \begin{pmatrix} \mathbf{0} & \sigma_k \\ -\sigma_k & \mathbf{0} \end{pmatrix} \tag{1.13}$$

The spinor $u(\vec{x}, t)$ has 4 components

$$u(\vec{x}, t) = \begin{pmatrix} u_1(\vec{x}, t) \\ u_2(\vec{x}, t) \\ u_3(\vec{x}, t) \\ u_4(\vec{x}, t) \end{pmatrix} \tag{1.14}$$

The Dirac equation with electromagnetic interaction will be derived in (5.5).

1.4.2 Field Theory and Quantum Physics

In the following it is convenient to introduce the commutator: p. 28f

$$[A, B] = AB - BA \tag{1.15}$$

and the anticommutator :

$$[A, B]_+ = AB + BA \tag{1.16}$$

For the creation and annihilation operators of bosons (particles with integer spin) holds:

$$[a(\vec{x}, t), a^*(\vec{y}, t)] = \delta(\vec{x} - \vec{y}), \quad [a(\vec{x}, t), a(\vec{y}, t)] = [a^*(\vec{x}, t), a^*(\vec{y}, t)] = 0 \tag{1.17}$$

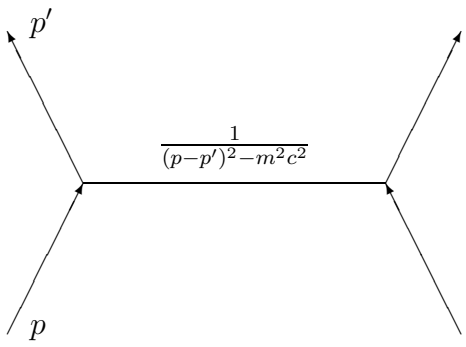

Figure 1.2: Diagramm for the exchange of a (scalar) particle with mass m

For fermions (particles with half-integer spin) the commutator $[,]$ is replaced by the anticommutator $[,]_+$.

• The complete form of the interaction s. (5.5) of a spin-$\frac{1}{2}$ particle with an electromagnetic field is: p. 30

$$L_{\text{Wechselwirkung}} = e^2\psi^*(\vec{x},t)\gamma_0\left(\gamma_0 A_0(\vec{x},t) - \sum_{k=1}^{3}\gamma_k A_k(\vec{x},t)\right)\psi(\vec{x},t) \tag{1.18}$$

where A_0 is the electric and $\vec{A}$ the magnetic (vector) potential, e is the charge and ψ the spinor field of the particle.

• An inner line of a quantum field of mass m, the so called propagator, gives rise to a factor: $1/(p^2 - m^2c^2)$,where p is the 4 momentum flowing through the inner line, see Figure 1 p. 32ff

The exchange of a particle with mass m gives therefore rise to a factor: p. 34

$$\frac{1}{(p-p')^2 - m^2c^2} \tag{1.19}$$

where p and p' are the 4 momentum before and after the exchange, respectively.

By Fourier transformation on obtains from this exchange term for small velocities – that is in the nonrelativistic limiting case – the following exchange potential:

$$V(r) \sim e^{-rmc/\hbar}/r \tag{1.20}$$

with $r = |\vec{x}\,|$

For the photon, which has invariant mass zero ($m = 0$) this yields the well known Coulomb potential, for mesons with mass $m \neq 0$ the formula yields the Yukawa potential with the range $r_0 = \hbar/(m\,c)$

The factor $\frac{1}{(p-p')^2}$,which occurs for the exchange of a photon gives rise to the term $1/\sin^4\frac{\theta}{2}$ in the formula for Rutherford scattering (Eq. 1.1).

• To the quantum correction of figure 1.16a (self energy) corresponds the divergent integral: p. 36

$$e^2 u^*(p)\gamma^0 \int d^4k \frac{-\gamma.q + m}{(q^2 - m^2)k^2} u(p), \qquad q = p - k \tag{1.21}$$

where $u(p)$ is the spinor of an electron with 4 momentum p ist, e is the charge and m the mass of the elctron; $\gamma.q$ is the scalar product of the γ matrices with the 4 momentum $q = p - k$, in units $\hbar = 1$ und $c = 1$. The term $q^2 - m^2$ in the denominator stems from the internal, that is virtual, electron line, the term k^2 from the internal photon line.

• Thomson had already calculated in 1904 the cross section for scattering of light off electrons. Of course he considered the flux not as flux of particles but as energy flux. It is however easy to convert the two fluxes into each other, since a single photon has the energy $\hbar\omega = h\nu$. p. 37

The Thomson cross section is:

$$\sigma = \frac{8\pi}{3} r_0^2, \quad r_0 = \frac{e^2}{m_e c^2}. \tag{1.22}$$

The quantity $r_0 = 2.8\ldots \cdot 10^{-15}$ m is called the classical electron radius: A charge distribution with extension r_0 leads to an electrostatic energy which is the rest energy of the electron, $mc^2 = 0.511\ldots$ MeV.

The graphs of figure 1.9 lead in the limit of vanishing photon momentum to the classical cross section 1.22, if one inserts for the charge e the classical charge. In this limiting case the higher order corrections do not contribute.

1.4.3 Quantum Physics and Errors

Though quantum mechanics is a deterministic theory, it predicts for the outcome of measurements only probabilities, that is the results of measurements show in general fluctuations. p. 39

A general results of mathematical statistics is: Is p the probability that an event x occurs and if there are N possibilities that the event may occur, then the probability distribution of the measured values x is the 'binomial distribution':

$$W_B(x) = \frac{N!}{x!(N-x)!} p^x \, (1-p)^{N-x} \tag{1.23}$$

In our example that means: Is N the total number of unstable particles and p the probability for the decay of a particle in one second, then $W_B(x)$ is the probability that in one second x decays are registered (under the assumption that each decay is registered).

For the important special case that p is very small and N very big the binomial distribution $W_B(x)$ becomes a 'Poisson distribution':

$$W_B(x) = \frac{N^x \, p^x}{x!} \tag{1.24}$$

Near the mean value $\bar{x} = pN$ the Poisson distribution can be approximated by a Gaussian (or normal) distribution:

$$W_G(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp^{(x-\bar{x})^2/(2\sigma^2)} \tag{1.25}$$

mit $\sigma = \sqrt{\bar{x}}$

σ is called the standard deviation. With $\bar{x} - \sigma \leq x \leq \bar{x} + \sigma$. If we want to calculate the probability p from the quotient of the observed decays, one has to take into account that this probability p is with a probability of 1/3 outside the interval $(\frac{x+\sigma}{N}, \frac{x-\sigma}{N})$.

The binomial distribution for the case $N = 1000$, $p = 0.1$ is shown in figure 1, solid line, also shown is the corresponding Gaussian distribution (dashed line).

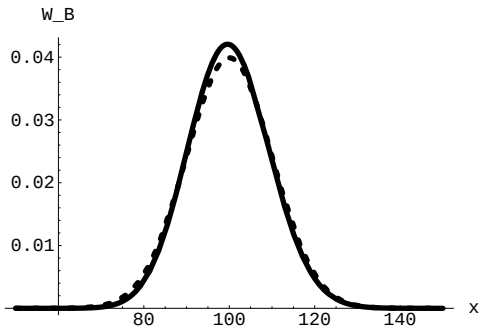

Figure 1.3: Binomial distribution (solid line) and Gaussian distribution (dashed line) for $N = 1000$, $p = 0.1$

Even if we can calculate exactly the probability p for an event, the results of measurements will nevertheless deviate from this value due to the above mentioned fluctuations. In order to compare theory with experiment on has to take into account this fluctuations. Since the standard deviation is $\sigma = \sqrt{pN}$ one obtains, with $pN \approx x$:

$$p = \frac{(x \pm \sqrt{x})}{N}. \tag{1.26}$$

The absolute standard error is thus: $\Delta p = \frac{\sqrt{x}}{N}$, the relative error which is in general the relevant quantity is then:

$$\frac{\Delta p}{p} = \frac{1}{\sqrt{x}}. \tag{1.27}$$

This shows that thew error decreases with the number of measured events, but unfortunately only with the square root. In order to increase the precision by a factor of 100 one needs 10 000 times more events.

1.5.1 Symmetries and Transformations

A *group* is a set G of elements,$a, b, \ldots$, for which there exists a composition law $a \circ b$ with p. 41ff

the following properties (the composition law is generally called *multiplication* and hence the reult of the composition *product*):

1. With a and b also the product $a \circ b$ is element of the group
2. There exists exactly one 1-Element with $1 \circ a = a \circ 1 = a$ for every a.
3. To each element a there exists exactly one *inverse element* designated as a^{-1}, with the property $a \circ a^{-1} = a^{-1} \circ a = 1$.
4. The composition is *associative*: $a \circ (b \circ c) = (a \circ b) \circ c$

Very often the composition, indicated above by $\circ$ is indicated just by juxtaposition, $a \circ b \equiv a\, b$.

• Matrices: A general $n \times n$ matrix is the quadratic scheme: p. 43ff

$$\mathbf{A} = (a_{ik}) = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{pmatrix} \tag{1.28}$$

The unit matrix is

$$1 = (\delta_{ik}) = \begin{pmatrix} 1 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ \vdots & & & \\ \dots & & & 1 \end{pmatrix} \tag{1.29}$$

The compostion law (product) for two matrices is:

$$\mathbf{A}\,\mathbf{B} = \begin{pmatrix} \sum_i a_{1i}b_{i1} & \sum_i a_{1i}b_{i2} & \sum_i a_{1i}b_{i3} & \dots & \sum_i a_{1i}b_{in} \\ \sum_i a_{2i}b_{i1} & \sum_i a_{2i}b_{i2} & \sum_i a_{3i}b_{i3} & \dots & \sum_i a_{2i}b_{in} \\ \sum_i a_{3i}b_{i1} & \sum_i a_{3i}b_{i2} & \sum_i a_{3i}b_{i3} & \dots & \sum_i a_{3i}b_{in} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \sum_i a_{ni}b_{i1} & \sum_i a_{ni}b_{i2} & \sum_i a_{ni}b_{i3} & \dots & \sum_i a_{ni}b_{in} \end{pmatrix} \tag{1.30}$$

The sum in $\sum_i$ goes from $i = 1$ to n. As we can see, the product of two $n \times n$ matrices is again an $n \times n$ matrix.

The matrix $\mathbf{A}$ *applied* to an n-dimensional vector v yields again a vector:

$$v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{pmatrix}; \quad \mathbf{A}\,v = \begin{pmatrix} \sum_i a_{1i}v_i \\ \sum_i a_{2i}v_i \\ \sum_i a_{3i}v_i \\ \vdots \\ \sum_i a_{ni}v_i \end{pmatrix} \tag{1.31}$$

The *adjoint matrix* $\mathbf{A}^*$ is defined as the complex conjugate and *transposed* of matrix $\mathbf{A}$.

$$\mathbf{A}^* = (a^*_{ki}) = \begin{pmatrix} a^*_{11} & a^*_{21} & a^*_{31} & \dots \\ a^*_{12} & a^*_{22} & a^*_{32} & \dots \\ a^*_{13} & a^*_{23} & a^*_{33} & \dots \\ \vdots & & & \\ \dots & & & a^*_{nn} \end{pmatrix} \tag{1.32}$$

For a real matrix the adjoint matrix is the transposed matrix.

A matrix $\mathbf{U}$ with real or complex components is called unitary if

$$\mathbf{U}^{-1} = \mathbf{U}^* \tag{1.33}$$

The matrix representing the rotation by an angle θ in the plane is

$$\mathbf{D}_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{1.34}$$

The rotated vector v' of $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ is:

$$v' = \begin{pmatrix} \cos\theta\, v_1 + \sin\theta\, v_2 \\ -\sin\theta\, v_1 + \cos\theta\, v_2 \end{pmatrix} \tag{1.35}$$

Two successive rotations by the angles θ und ϕ respectively yield the matrix:

$$\begin{aligned} D_\phi\, D_\theta &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta\ \cos\phi) - \sin\theta\, sin\phi & \sin\theta\ \cos\phi + \cos\theta\ \sin\phi \\ -\sin\theta\ \cos\phi - \cos\theta\ \sin\phi & \cos\theta\ \cos\phi - \sin\theta\ \sin\phi \end{pmatrix} \\ &= \begin{pmatrix} \cos(\theta+\phi) & \sin(\theta+\phi) \\ -\sin(\theta+\phi) & \cos(\theta+\phi) \end{pmatrix} \end{aligned} \tag{1.36}$$

The product is the rotation matrix with the angle $\theta + \phi$ (that the resulting angle is just the sum of the two angels is special for the plane).

For the rotation matrices holds:

$$\mathbf{D}^* = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \mathbf{D}_{-\theta} = \mathbf{D}^{-1} \tag{1.37}$$

This property is characteristic for rotations, it guarantees that the length of a vector and the angle between two vectors is not changed.

1.5.2 The Miracle of Spin

• Stricly speaking, the operators of angular momentum are the generators of rotations in p. 45f

three-dimensional space multiplied by $\hbar$.

A rotation around the z axis by the angle ϕ can be represented with the help of the generators by:

$$e^{i\phi \mathbf{L}_z} \tag{1.38}$$

The exponential function of an operator can e.g. be defined by a power series:

$$e^{i\mathbf{L}_z\phi} = 1 + i\phi\mathbf{L}_z\phi + (i\phi\mathbf{L}_z)^2/2 + (i\phi\mathbf{L}_z)^3/6 + \ldots \tag{1.39}$$

The generators of $SU(2)$, $\mathbf{s_1},\ \mathbf{s_2},\ \mathbf{s_3}$ are given by the Pauli matrices σ:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1.40}$$

that is: $\mathbf{s_k} = \frac{1}{2}\sigma_k, \quad k = 1 \ldots 3.$

One checks directly the commutation relations:

$$\mathbf{s_1}\,\mathbf{s_2} - \mathbf{s_2}\,\mathbf{s_1} = i\mathbf{s_3} \tag{1.41}$$

The rotation matrix for a spinor around the z axis is given by:

$$e^{i\mathbf{s}_3\phi} = \begin{pmatrix} \cos\frac{\phi}{2} + \sin\frac{\phi}{2} & 0 \\ 0 & \cos\frac{\phi}{2} - \sin\frac{\phi}{2} \end{pmatrix} \tag{1.42}$$

For $\phi = 360°$ one has $\cos(\phi/2) = -1$ and $\sin(\phi/2) = 0$, that is for a rotation by $\phi = 360°$ one obtains that the rotation matrix of a spinor around the full circle is the negative of the unit matrix! This means under a rotation by a full circle a spinor does not come back into its old state but changes its sign. The consequences of this counterintuitive result have been checked in experiments.

1.6 The Discovery of the Positron and the 'Mesotron'

Let a charged particle of mass m charge e move with momentum $\vec{p}$ in an homogeneous magnetic field $\vec{B}$. The following equation of motion holds: p. 53

$$\frac{d\vec{p}}{dt} = \frac{e}{c}[\vec{v} \times \vec{B}], \quad |\vec{p}| = \text{konstant} \tag{1.43}$$

For velocities perpendicular to the magnetic field the reulting solution is a circular trajectory with radius

$$R = c|\vec{p}|/(eB). \tag{1.44}$$

Therefore the momentum can be determined from the radius R.

1.7 Early Accelerators

For $v \ll c$ follows from 1.44 p. 59

$$R = \frac{mvc}{eB}, \tag{1.45}$$

and for the frequency $f = v/(2\pi R)$

$$f = \frac{eB}{2\pi mc}, \tag{1.46}$$

that is the frequency is in nonrelativistic approximation independent of velocity and hence of energy.

The following relation holds for all velocities:

$$v = pc^2/E \tag{1.47}$$

where E is the total energy of the particle. Hence the frequency is in general:

$$f = \frac{ecB}{2\pi E}, \tag{1.48}$$

that is it decreases with increasing energy (Note: in nonrelativistic approximation the total energy is just the rest energy).

Chapter 2

The Great Leap Forward

2.4 Successes and Failures of Quantum Field Theory

The interaction term of a nucleon with a *pi* meson is: p. 75

$$L_{\text{interaction}} = g_{\pi N}\psi^*(\vec{x},t)\gamma_0\gamma^5\psi(\vec{x},t)\phi(\vec{x},t) \tag{2.1}$$

where $g_{\pi N}$ is the *pi*-meson-nucleon coupling and ϕ the quantum field of the *pi*-meson, for the matrix γ^5 s. (2.2). The coupling constant has approximately the value $g^2_{\pi N}/(4\pi) \approx 14$, which is very large with the electromagnetic coupling $e^2/(4\pi) \approx 1/137$ in QED.

The so-called γ^5 matrix is the product of the 4 γ matrices introduced in (1.13):

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{2.2}$$

It occurs in the interaction term, since the pi meson is a pseudoscalar particle, that it its quantum field changes sign under space reflection.

2.7 More and More New Particles

The x-axis of figure 2.13 is the 'invariant mass' of the two produced pi-mesons $m_{\pi^+\pi^-}$. p. 87

$$m^2_{\pi^+\pi^-}c^2 = (p'_{\pi^+} + p'_{\pi^-})^2 \tag{2.3}$$

s. (1.10)

2.8 The Surprises of the Weak Interaction

2.8.1 Digression: Right- and Left-Handed Particles

The relation between Weyl- and Dirac spinors is given through the γ_5 matrix (2.2): p. 92f

$$\chi_{\text{rechts}} = \frac{1+\gamma^5}{2}\psi_{\text{Dirac}}, \quad \chi_{\text{links}} = \frac{1-\gamma^5}{2}\psi_{\text{Dirac}} \tag{2.4}$$

χ are left and right-handed Weyl spinors respectively and ψ is a Dirac spinor.

2.8.2 And Now Back to Weak Interactions

In weak interactions only left handed fermions(and right handed antifermions) participate, p. 94f
that is the factor $(1-\gamma^5)\psi$ occurs in the interaction. This implies for the interaction term of mu-decay ($\mu \to e + \nu_e + \nu_\mu$):

$$\mathcal{L}_{\text{interaction}} = G_F \sum_{\rho=0}^{3} \left(\psi_e^* \gamma^0 (1-\gamma^5)\gamma^\rho \psi_{\nu_e}\right)\left(\psi_{\nu_\mu}\gamma^0(1-\gamma^5)\gamma_\rho\psi_\mu\right) \tag{2.5}$$

where the indices at the fields ψ indicate which particles they describe, ψ_{ν_e} e.g. is the field of the electron neutrino and its antiparticle; G_F is the weak coupling (Fermi coupling constant)

The expression $\psi_e^*\gamma^0(1-\gamma^5)\gamma^\rho\psi_{\nu_e}$ is called a '*charged weak $V-A$ current*'. It is called charged since it changes the charge (from neutrino to electron), weak since it occurs in the weak interaction, and $V-A$ since the part $\psi_e^*\gamma^0\gamma^\rho\psi_{\nu_e}$ behaves as a normal vector (V) and the part $\psi_e^*\gamma^0\gamma^5\gamma^\rho\psi_{\nu_e}$ like an axial vector (A).

• For CP violation one makes the ansatz: p. 96

$$K_S = pK^0 + q\bar{K}^0, \quad K_L = pK^0 - q\bar{K}^0 \tag{2.6}$$

where $p/q-1$ is a measure for the CP violation. The measured value for it is very small $p/q-1 \approx 0.001$. This implies that CP symmetry is only very weakly violated.

Chapter 3

Up by Their Own Bootstraps

3.2 Scattering Amplitudes

The cross section is determined from the scattering amplitude T as p. 100

$$\sigma = \frac{1}{F}|T|^2 d\Gamma, \tag{3.1}$$

where F is the particle flux and $d\Gamma$ the phase space factor.

In the rest system of the target particle holds:

$$F = m_t(E_s - m_s), \tag{3.2}$$

where E_s and m_s are energy and mass of the scattered particle, $m_t$is the mass of the target particle.

The phase space factor $d\Gamma$ is the measure of all components of the 4 momenta of the final states with the restriction that the squares of the single momenta are the rest masses of the particles and that the total final momentum is equal to the total initial momentum p_{tot}. For N particles in the final stae one has:

$$d\Gamma^N = \delta^4\left(\sum_{i=1}^{N} p_{(i)} - p_{\text{tot}}\right)\prod_{i=1}^{N} d^4p_{(i)}\,\delta(p_{(i)} - m^2) \tag{3.3}$$

• Very sketchily one can illustrate the consequences of unitarity as follows: Unitarity implies that the product of the S matrix with its adjoint is equal to the unit matrix. p. 105

$$S^\dagger S = 1 \tag{3.4}$$

If the S matrix is equal to the unit matrix this means that the state remains unchanged by the scattering process, that is nothing has happened at all. If we subtract the unit matrix from the S matrix we obtain its interesting part, the scattering amplitude, normally called T matrix

$$T = (S-1)/i$$

(the division by the imaginary unit i is pure convention). The cross section is proportional to $|T|^2$, as noted above.

From unitarity (3.4) follows:

$$1 = S^\dagger S = (1+iT)(1-iT^\dagger) = 1 + i(T - T^\dagger) + TT^\dagger \tag{3.5}$$

oder

$$TT^\dagger = \frac{1}{i}(T - T^\dagger) \tag{3.6}$$

From the last equation follows the *optical theorem*, it states that the total cross section is equal to the imaginary part of the scattering amplitude in forward direction. For actual calculations one has of course also to take into account the phase-space factor $d\Gamma$.

• In the following be W the total energy and θ the scattering angle in the center of momentum frame, that is in a frame in which the sum of the 3 momenta of the colliding particles is zero. p. 102

For pi meson nucleon scattering one obtains for energies and momenta of the single particles (book, Figure 3.2.a):

$$|\vec{p}_\pi|^2 = |\vec{p}_p|^2 = \frac{1}{4W^2}\sqrt{[(W^2-(m_p+m_\pi)^2][W^2-(m_p-m_\pi)^2]} \tag{3.7}$$

$$E_p = \frac{1}{2W}\sqrt{W^2+m_p^2-m_\pi^2}, \quad E_\pi = \frac{1}{2W}\sqrt{W^2+m_\pi^2-m_p^2}. \tag{3.8}$$

The Mandelstam variables are:

$$s = (p_{\pi^-} + p_p)^2 = (p_{\pi^-}{}' + p_p{}')^2 = W^2, \tag{3.9}$$
$$t = (p_{\pi^-} - p_{\pi^-}{}')^2 = (p_p - p_p{}')^2 = -2\vec{p}_\pi{}^2(1 - \cos\theta) \tag{3.10}$$

For proton-antiproton annihilation into two pi mesons holds (figure 3.2 b):

$$|\vec{p}_\pi| = \frac{1}{2W}\sqrt{W^2 - 4m_\pi^2}, \quad |\vec{p}_p| = \frac{1}{2W}\sqrt{W^2 - 4m_p^2} \tag{3.11}$$
$$E_p = E_\pi = \frac{W}{2} \tag{3.12}$$

The Mandelstam variables for this annihilation process are:

$$t = (p_p + p_{\bar{p}})^2 = (p_{\pi^-} - p_{\pi^+})^2 = W^2 \tag{3.13}$$
$$s = (p_p - p_{\pi^+})^2 = (p_{\bar{p}} - p_{\pi^-})^2 \tag{3.14}$$
$$= m_p^2 + m_{pi}^2 - \frac{1}{2}\Big(W^2 - \sqrt{[W^2 - 4m_p^2][W^2 - 4m_\pi^2]}\cos\theta\Big)$$

The graph of book, Figure 3.2.c has for pi-meson nucleon scattering the singularity (pole), compare (1):

$$\frac{1}{s - m_n^2} = \frac{1}{(p_\pi + p_p)^2 - m_n^2} = \frac{1}{W^2 - m_n^2}. \tag{3.15}$$

For the annihilation of a proton and an antiproton the pole term is, see 3.13):

$$\frac{1}{(p_p - p_{\pi^+})^2 - m_n^2} = \frac{1}{m_p^2 + m_\pi^2 - \frac{1}{2}\Big(W^2 - \sqrt{[W^2 - 4m_p^2][W^2 - 4m_\pi^2]}\cos\theta\Big) - m_n^2} \tag{3.16}$$

where now W refers to the center of momentum system of the proton and antiproton.

3.4 Rigorous Theorems and Complex Angular Momenta

From axiomatic field theory follows for the total cross section of two hadrons the rigorous p. 107

Froissart-Martin limit:

$$\sigma \leq \frac{\pi}{m_\pi^2}\Big(\log(W^2/W_0^2)\Big)^2, \tag{3.17}$$

W_0 is not determined in the theorem.

• The decisive step for the analytic continuation of the angular momenta consists in converting the infinite sum over partial waves into an integral and then deforming the integration path in order to obtain a finite sum over pole terms. p. 108

• The Veneziano amplitude for scattering of two equal particlesof mass m is: p. 109

$$A(s,t) = B(-\alpha(s), -\alpha(t)) + B(-\alpha(s), -\alpha(u))B(-\alpha(t), -\alpha(u)) \tag{3.18}$$

where the Euler Beta function $B(x,y)$ is given by the Gamma functions:

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} \tag{3.19}$$

and $u = 4m^2 - s - t$, and $\alpha(s) = \alpha.\, s + \alpha(0)$ is the Regge trajectory

This amplitude exhibits poles if the trajectories take integer values, e.g. if $\alpha(s) = n$, since the Gamma function has a pole for negative integer values. The amplitude (3.18) has also the correct asymptotic behavior as can be checked by using the Stirling formula for large z for the Gamma function:

$$\Gamma(z) \sim e^{-z} z^{z-1/2}\sqrt{2\pi} \tag{3.20}$$

Chapter 5

On the Path to the Standard Model

5.1 The Master of the Gauge

If a wave function of quantum physics is multiplied by a phase factor, that is a constant complex number with modulus 1, it describes still the same physical state. The phase factors $e^{i\lambda}$, λ real, are by definition the elements of the group $U(1)$, therefore a more learned and mathematically precise formulation for the statement on the wave function made above is: *Quantum mechanics is invariant under transformations of the group* $U(1)$. p. 134

Weyl regarded it as highly implausible that the λ in the phase factor has to be strictly constant, even if the wave function is extended over a very large region of space. In analogy to general relativity he called this property a 'parallelism at a distance' (*Fernparallelismus)* which he rejected. Here therefore *postulated* that quantum mechanics remains invariant, even if the phase factor is space and time dependent, that is if the wave function is multiplied by $e^{ie\lambda(\vec{x},t)}$.

The rejection of the parallelism at a distance implies: The wave function $u(\vec{x},t)$ and the 'gauged' wave function $e^{i\lambda(\vec{x},t)}u(\vec{x},t)$ must describe the same physical state. At a first glance this seems wrong: The wave function $u(\vec{x},t)$ of a free (that is noninteracting) spin

$\frac{1}{2}$ particle satisfies the linear Dirac equation (1.11):

$$\left(\gamma_0\frac{\partial}{\partial x_0}+\gamma_1\frac{\partial}{\partial x_1}+\gamma_2\frac{\partial}{\partial x_2}+\gamma_3\frac{\partial}{\partial x_3}-mc^2\right)u(\vec{x},t)=0 \qquad (x_0=ct) \tag{5.1}$$

It plays in this context no role that the quantities γ^ρ are 4×4-matrices; if you dislike matrices it does for these considerations no harm to take the γ matrices as numbers

If the gauge transformation

$$u(\vec{x},t)\rightarrow u'(\vec{x},t)=e^{ie\lambda(\vec{x},t)}u(\vec{x},t) \tag{5.2}$$

is performed the transformed wave function $u'(\vec{x},t)$ does no longer satisfy the Dirac equation (5.1), that is inserted in the Dirac equation u' yields no longer zero. This can be easily checked by inserting u' into equation (5.1) and applying Leibniz's rule:

$$\begin{aligned}
&\left(\gamma_0\frac{\partial}{\partial x_0}+\gamma_1\frac{\partial}{\partial x_1}+\gamma_2\frac{\partial}{\partial x_2}+\gamma_3\frac{\partial}{\partial x_3}-m\right)u'(\vec{x},t) &(5.3)\\
=\;&\Big(\gamma_0[\frac{\partial}{\partial x_0}u(\vec{x},t)+i\,e\,u(\vec{x},t)\frac{\partial}{\partial x_0}\lambda(\vec{x},t)]\\
+\;&\gamma_1[\frac{\partial}{\partial x_1}u(\vec{x},t)+i\,e\,u(\vec{x},t)\frac{\partial}{\partial x_1}\lambda(\vec{x},t)]\\
+\;&\gamma_2[\frac{\partial}{\partial x_2}u(\vec{x},t)+i\,e\,u(\vec{x},t)\frac{\partial}{\partial x_2}\lambda(\vec{x},t)]\\
+\;&\gamma_3[\frac{\partial}{\partial x_3}u(\vec{x},t)+i\,e\,u(\vec{x},t)\frac{\partial}{\partial x_3}\lambda(\vec{x},t)]-m\,u(\vec{x},t)\Big)e^{i\lambda(\vec{x},t)}\\
=\;&\Big(\gamma_0\frac{\partial}{\partial x_0}\lambda(\vec{x},t)+\gamma_1\frac{\partial}{\partial x_1}\lambda(\vec{x},t)+\gamma_2\frac{\partial}{\partial x_2}\lambda(\vec{x},t)+ &(5.4)\\
&\qquad\gamma_3\frac{\partial}{\partial x_3}\lambda(\vec{x},t)\Big)i\,e\,u(\vec{x},t)\neq 0
\end{aligned}$$

In order to get rid of this term we must introduce an additional field and modify the free

Dirac equation to:

$$\Big(i\gamma_0[\frac{\partial}{\partial x_0}+ieA^0]+\gamma_1[\frac{\partial}{\partial x_1}+ieA^1]+\gamma_2[\frac{\partial}{\partial x_2}+ieA^2] \\ +\gamma_3[\frac{\partial}{\partial x_3}+ieA^3]-m\Big)u(\vec{x},t)=0 \tag{5.5}$$

If the wave function is gauge transformed according to 5.2 and *simultaneously* the newly introduced 4-vector field $(A^0,\vec{A})$ is transformed according to:

$$A^\rho(\vec{x},t)\rightarrow A'^\rho(\vec{x},t)=A^\rho(\vec{x},t)-\frac{\partial}{\partial x_\rho}\lambda(\vec{x},t),\quad \rho=0\ldots3, \tag{5.6}$$

on can check easily that $u'(\vec{x},t)$ and $A'(\vec{x},t)$ are solutions of eq. 5.5, if this is also the case for $u(\vec{x},t)$ and $A(\vec{x},t)$.

The transformation 5.6 is just the time honored gauge transformation of the 4 potential in electrodynamics, consisting of the electric potential A^0 and the magnetic vector potential $\vec{A}$, the equation (5.5) is the Dirac equation for a charged particle in an electromagnetic field. This is the equation with the help of which Dirac could explain the fine structure of the hydrogen atom and the correct gyromagnetic ratio of the electron.

We see: If we *postulate* the invariance of the Dirac equation under the gauge transformation (5.2), the the existence of the electromagnetic field and the form of the equation (5.5) describing its interaction with an electrically charged particle is a consequence of this postulate.

The electric and magnetic *field strength* is not influenced by the gauge transformation of the potentials (5.6) As an example consider the 1 component of the electric $(\vec{E})$:

$$E'^1=\frac{\partial}{\partial x_1}A'^0-\frac{\partial}{\partial x_0}A'^1=\frac{\partial}{\partial x_1}A^0-\frac{\partial^2}{\partial x_1\partial x_0}\lambda-\frac{\partial}{\partial x_0}A^1+\frac{\partial^2}{\partial x_0\partial x_1}\lambda=E^1 \tag{5.7}$$

Let us summarize. In quantum physics $U(1)$ symmetry holds: If the wave function is multiplied by a constant phase factor, the physical state remains unchanged. If one

'gauges this symmetry', that is one postulates that invariance still holds if the phase factor is space-time dependent then this postulate of gauge invariance has far reaching consequences: There must exist a new field, the so called gauge field and the interaction of this new field with the particles is also determined – at least in its minimal form.

5.2 New Dimensions for the Gauge

The transformations of the group $SU(2)$ are generated by the Pauli matrices σ_k (1.40), p. 138
that is an isospinor χ is transformed according to:

$$\chi \to \chi' = e^{ig\sum_k \lambda_k \sigma_k}\chi \tag{5.8}$$

where the sum runs from $1 \ldots 3$, g is the $SU(2)$ gauge coupling.

The group $SU(2)$ is *gauged* by making the three real coefficients $\lambda_1, \lambda_2, \lambda_3$ space-time dependent:

$$\chi \to \chi' = e^{i\sum_k \lambda_k(\vec{x},t)\sigma_k}\chi \tag{5.9}$$

Because there are *three* generators in $SU(2)$, one has to introduce in the process of gauging this symmetry *three* gauge-4-potentials, namely: $A^0_{(i)}, A^1_{(i)}, A^2_{(i)}, A^3_{(i)}), \quad i = 1, 2, 3.$

The analogues of the electric and magnetic fields are in such a 'non-Abelian' gauge theory more complicated than in electrodynamics. For the 1-component of the first of three 'electric' fields one obtains:

$$E^1_{(1)} = \frac{\partial}{\partial x_1}A^0_{(1)} - \frac{\partial}{\partial x_0}A^1_{(1)} + ig(A^1_{(2)}A^0_{(3)} - A^0_{(2)}A^1_{(3)}) \tag{5.10}$$

The additional nonlinear term $ig(A^1_{(2)}A^0_{(3)} - A^0_{(2)}A^1_{(3)})$. is responsible for the interaction of the gauge bosons (Book, Figure 5.34 und b) among themselves.

5.3 Spontaneous Symmetry Breaking

As mentioned in the book we combine the two fields F_1 and F_2 to a complex field: p. 146

$$F(\vec{x},t) = F_1(\vec{x},t) + iF_2(\vec{x},t). \tag{5.11}$$

The static field energy is then easily expressed as:

$$V(F) = MF(\vec{x},t)^* F(\vec{x},t) + g\Big(F(\vec{x},t)^* F(\vec{x},t)\Big)^2 \tag{5.12}$$

where the asterisk * indicates complex conjugation.

The complex field F (5.11) can be decomposed into modulus $\eta(\vec{x},t)$ and the phase (argument) $\zeta(\vec{x},t)$:

$$F(x) = \frac{1}{\sqrt{2}}(\eta(\vec{x},t) + \sqrt{2}\, v) \exp\Big[i\frac{\zeta(\vec{x},t)}{\sqrt{2v}}\Big] \tag{5.13}$$

where we have shifted the field by $v = \sqrt{-M/(2g)}$ in order to have the minimum of the static energy (5.12) at $\eta = 0$. From the decomposition (5.13) follows:

$$F(\vec{x},t)^* F(\vec{x},t) = \frac{1}{2}(\eta(\vec{x},t) + \sqrt{2}\, v)^2 \tag{5.14}$$

and we obtain after an elementary calculation:

$$V = -M\eta^2 + \eta^3\sqrt{-Mg}\, + \frac{g}{4}\eta^4 - \frac{M^2}{2g}. \tag{5.15}$$

In this expression for the static field energy the field ζ does no longer appear, especially there is no mass term for this field. Indeed, the field $\zeta(\vec{x},t)$ is the field of the massless Goldstone boson. The ζ field appears, however in the kinetic contribution to the field energy and also shows a characteristic interaction with the η field. The field η has the positive mass term $-M\eta^2$ (note that M is negative).

If we perform a perturbative expansion in the coupling g we find the following particle content:

1. A massless boson, the Goldstone boson described by the field ζ
2. A boson with mass $\sqrt{-M}$, the so called Higgs boson, described by the field η.

5.4 The Higgs-Kibble Dinner

If the $U(1)$ symmetry of the considered model is *gauged* by making the phase factor transforming the field F space-time dependent then we have to add the gauge potential A_μ. This gives an additional term to the static field energy: p. 147

$$e^2 \sum_{\mu=0}^{3} A_\mu(\vec{x},t)A^\mu(\vec{x},t)F(\vec{x},t)^*F(\vec{x},t), \tag{5.16}$$

i.e. we have instead of the expression (5.12):

$$V(F) = MF^*F + g\Big(F^*F\Big) + e^2 \sum_{\mu} A_\mu A^\mu F^*F \tag{5.17}$$

where we have written F for $F(\vec{x},t)$ etc. .

If we substitute F by η and ζ according to (5.13)we obtain:

$$V = -M\eta^2 + \eta^3\sqrt{-Mg} + \frac{g}{4}\eta^4 - \frac{M^2}{2g} + e^2 \sum_{\mu} A_\mu A^\mu \left(\frac{1}{2}\eta^2 + \sqrt{\frac{-M}{g}}\eta + \frac{-M}{2g}\right). \tag{5.18}$$

Now we have also a quadratic term in the gauge potential, namely $\frac{-M}{2g}\, e^2 \sum_\mu A_\mu A^\mu$. This is a mass term and the gauge field has acquired the mass

$$m_A = \frac{-M}{2g}\, e^2.$$

In the equations there occurs still formally the field ζ of the Goldstone boson, but now we can 'gauge it away': If we make the gauge transformation, see (5.2,5.6) we obtain:

$$F \to e^{-i\zeta/\sqrt{2v}}F, \quad A_\mu \to A_\mu + \frac{\partial}{\partial x^\mu}\frac{\zeta}{\sqrt{2ve}}. \tag{5.19}$$

The gauge transformation eliminates the field ζ completely from eq. (5.13), but physics is not changed, since the performed transformation is a gauge transformation.

5.6 Better Counters, Better Accelerators, and Better Beams

The total energy W in the center of mass (center of momentum) system (CMS) is given by: $W^2 = s = (p_1 + p_2)^2$. In the lab system (target at rest) the 4-momentum vector of the target is $p_1 = (m_1, \vec{0}\,)$ that is we have for the total energy: p. 152

$$W = m_1^2 + m_2^2 + 2\sqrt{m_1 E_2}. \tag{5.20}$$

The total energy in the CMS increases in the lab system only with the square root of the energy of the accelerated particle.

If both particles are accelerated, with opposite 3-momentum, $\vec{p}_1 = -\vec{p}_2$, then the total energy in the CMS increases lenearly:

$$W = E_1 + E_2. \tag{5.21}$$

5.7 The Electron Microscopes of Particle Physics

In an elastic collision of two particles the maximum momentum transfer is given by: p. 157

$$|Q| \leq 2|\vec{p}^{\,*}| \tag{5.22}$$

where $\vec{p}^{\,*}$, the momentum in the CMS, is given by s. (3.7):

$$|\vec{p}^{\,*}| = \sqrt{\frac{(W^2 - (m_1 + m_2)^2)(W^2 - (m_1 - m_2)^2)}{4W^2}}. \tag{5.23}$$

Here W is the total energy in the CMS.

For relativistic particles one can neglect the small mass terms and one has approximately

$$|Q| \leq W. \tag{5.24}$$

This is the reason why one needs high energies in order to resolve small structures.

5.8 Deep Inelastic Scattering

For the analysis of deep inealstic scattering the optical theorem is very important. The lower part of the graph of book, Figure 5.15 can be regarded as the scattering of a virtual photon on protons, the upper part is known from quantum electrodynamics. Therefore on can say that in deep inelastic scattering one measures the total cross sections for the scattering of a virtual photon on a proton. This total cross section is proportional to the imaginary part of the forward scattering amplitude which in turn can be analyzed theoretically. p. 162

The details of the treatment of deep inelastic scattering are very complicated. Because of spin and the different currents (electromagnetic and weak) one has three structure functions. In order to determine them one has to make detailed measurements with charged and neutral leptons (electrons, muons, neutrinos, and their antiparticles).

Chapter 6

The Standard Model of Particle Physics

6.3 Weak Currents

The doublet of quarks , corresponding to the doublet of leptons (Fig 6.2) is: p. 171f

$$\begin{pmatrix} u \\ d\,\cos\theta_C + s\,\sin\theta_C \end{pmatrix} \tag{6.1}$$

To the doublet of the muon and ist antineutrino corresponds:

$$\begin{pmatrix} c \\ s\,\cos\theta_C - d\,\sin\theta_C \end{pmatrix} \tag{6.2}$$

Here u, d, s, c stands for up, down, strange and charmed quark, θ_C is the so called Cabbibo angle.

6.5 Running Coupling and Asymptotic Freedom

The running of the coupling follows from the equation: p. 183f

approximate energy range μ (GeV)	number of active quark flavors n_f	$\Lambda^{(n_f)}$ (MeV)
> 200	6	88 ± 11
$10 - 200$	5	208 ± 25
$3 - 10$	4	288 ± 30
< 3	3	326 ± 30

Table 6.1: Numerical values of the active flavors n_f and of the ACD scale $\Lambda_{\rm QCD}$

$$\mu \frac{\partial \alpha_s}{\partial \mu} = 2\beta(\alpha_s) \tag{6.3}$$

where μ is the energy scale, that is μ is inversely proportional to the distance scale. If the quantity $\beta(\alpha_s)$ is negative then α_s decreases with increasing μ and correspondingly decreases with decreasing distance. The theory is then called asymptotically free.

In perturbation theory on can calculate β:

$$2\beta(\alpha_s) = -\frac{\beta_0}{2\pi}\alpha_s^2 - \frac{\beta_1}{4\pi^2}\alpha_s^3 - \frac{\beta_2}{64\pi^3}\alpha_s^4 - \dots \tag{6.4}$$

with

$$\begin{aligned} \beta_0 &= 11 - \frac{2}{3}n_f \\ \beta_1 &= 51 - \frac{19}{3}n_f \\ \beta_2 &= 2587 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2 \end{aligned}$$

where n_f is the number od 'active flavors' (see table 6.1)

Taking into account only the lowest order perturbative contribution, β_0, one obtains the transparent relation:

$$\frac{d\alpha_s}{\alpha_s^2} = \frac{\beta_0}{2\pi}\frac{d\mu}{\mu}. \tag{6.5}$$

The solution to this differential equation is

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \log\left(\frac{\mu^2}{\Lambda^2_{\rm QCD}}\right)} \tag{6.6}$$

where the integration constant $\Lambda_{\rm QCD}$ is a quantity typical for QCD, see table 6.1

The use of a running coupling corresponds to a resummation of all powers of $\alpha(\mu_1)\log(\mu_2^2/\mu_1^2)$. Unfortunately this 'renormalization group improvement' depends not only on the renormalization scale μ, but also in many cases on the choice of the renormalization scheme. For many cases the most convenient scheme is the so called $\overline{MS}$ (speak MS-bar) scheme. MS stands for *Minimal Subtraction*.

• For QED one has in lowest order perturbation theory: $\beta_0^{\rm QED} = -\frac{4}{3}$. Since $\beta(\alpha) > 0$ the running coupling increases with increasing energy scale. QED is not an asymptotically free theory like QCD! The natural lower limit for the energy scale in QED is the electron mass m_e. Restricting to lowest order β_0 we obtain : p. 185

$$\alpha(\mu) = \frac{\alpha(m_e)}{1 + \frac{\beta_0^{\rm QCD}}{4\pi}\alpha(m_e)\log\left(\frac{\mu^2}{m_e^2}\right)} \tag{6.7}$$

with $\alpha(m_e) = \alpha = \frac{1}{137}$.

The expression (6.7) has a singularity at $\mu = m_e e^{\frac{3\pi}{2\alpha}}$. This is the so called Landau Pole, it occurs at an energy of ca. 10^{277} GeV ! Compared to this energy the Planck scale is tiny. .

The radiative corrections of QED alone lead at the scale of the Z^0 mass to an increase of the running electromagnetic coupling α by ca 2%.

6.7 Quantum Chromodynamics on the Lattice

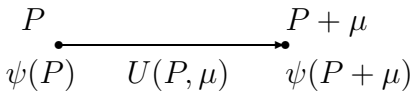

Figure 6.1: The fields on a lattice. The Fermion fields are assigned to points, the gauge fields to links between neighboring points.

Be P a lattice point and $P+\mu$ a neighboring point in μ direction; be a the lattice spacing, that is the distance between two neighboring lattice points. If P has the coordinates $(0,0,0,0)$ and if $\mu = 3$, then $P+\mu$ has the coordinates $(0,0,a,0)$. Be $\psi(P)$ a fermion field at a lattice point P and $U(P,\mu)$ a field corresponding to a *link* of P to $P+\mu$. U is an element of the gauge group, for electrodynamics, where the gauge group is $U(1)$, one has: p. 190

$$U(P,\mu) = \exp[iaA_\mu(P)]. \tag{6.8}$$

where A_μ is the electromagnetic potential.

The essential part of the interaction on the lattice is given at one hand by the pure gauge part:

$$\sum_{\mu,\nu=1}^{4} U(P,\mu)U(P+\mu,\nu)U(P+\mu+\nu,-\nu)U(P+\mu,-\mu), \tag{6.9}$$

on the other hand by the fermionic (matter) part

$$\sum_{\mu=1}^{4} \psi^*(P+\mu)\gamma_\mu U(P,\mu)\psi(P). \tag{6.10}$$

In the limit $a \to 0$ the classical limit of these expressions is just the classical electromagnetic interaction.

Lattice QCD is defined on an Euclidian space-time continuum. This is obtained from the usual Minkowskian space time continuum by replacing the time coordinate by an

imaginary quantity (that is the time honored ict metric):

$$ct = x_0 \rightarrow -ix_4, \quad x_4 \text{ reell} \tag{6.11}$$

The Minkowskian metric tensor (1.6) is replaced by the Euclidian one, that is Kronecker deltas:
$g_{\mu\nu} \rightarrow -\delta_{\mu\nu}$

Chapter 7

Storm Clouds or the Dawn of a New Physics?

7.3 **The Grand Unification**

One family of elementary particles consists of 15 fermions: 12 quark states, the lepton, antilepton and neutrino. Let us for simplicity concentrate on the lowest family, that of u and d quarks, the electron e, and the electron-neutrino ν_e. The 12 quark states are the respective 3 color states of the u and the d quarks and the corresponding antiparticles. p. 221f

In the model of Georgi and Glashow ($SU(5)$) the three 3 u and 3 d quarks, the 3 u antiquarks und the positron form the basis of a 10 dimensional representation. The d antiquarks, the electron and the neutrino form the basis of a 5 dimensional represntation.

$$\begin{aligned} (\mathbf{10}) &= \{u_R, u_B, u_w, d_R, d_B, d_W, \bar{u}_R, \bar{u}_B, \bar{u}_W, \bar{e}\} \\ (\mathbf{5}) &= \{\bar{d}_R, \bar{d}_B, \bar{d}_W, e, \nu\} \end{aligned}$$

where the indices $_{R\ B\ W}$ stand for the 3 colors of color-$SU(3)$ and the bar $^-$ denotes antiparticles.

7.4 **Supersymmetry**

The simplest supersymmetric algebra is generated by the following generators: the energy operator $\mathbf{P}_0$, that is the translation operator in time, the momentum operators $\vec{\mathbf{P}}$, the translation operators in space and fermionic operators Q_α with the anticommutation relations: p. 223

$$\begin{aligned} [\mathbf{Q}_\alpha, \bar{\mathbf{Q}}_{\dot{\beta}}]_+ &= 2\mathbf{P}_0 - 2\vec{\sigma}_{\alpha\dot{\beta}}\vec{\mathbf{P}} \\ [\mathbf{Q}_\alpha, \mathbf{Q}_\beta]_+ &= [\bar{\mathbf{Q}}_{\dot{\alpha}}, \bar{\mathbf{Q}}_{\dot{\beta}}]_+ = 0 \\ [P_m, Q_\alpha]_- &= [P_m, \bar{Q}_{\dot{\alpha}}]_- = 0 \\ [P_m, P_n]_- &= 0 \end{aligned} \tag{7.1}$$

with $m, n = 0 \ldots 3$, $\alpha, \dot{\alpha} \ldots = 1, 2$.

7.7 Silent Strings

The equation for the displacement η of a vibrating string is: p. 234

$$\left(\frac{d^2}{dx^2} - \frac{1}{v^2} \frac{d^2}{\partial t^2} \right) \eta(x,t) = 0 \tag{7.2}$$

where v is determined by the linear mass density μ and the tension T of the string: $v = \sqrt{T/\mu}$.

This wave equation 7.2 has the most general solution (d'Alembert 1747):

$$\eta(x,t) = f(x - vt) + g(x + vt) \tag{7.3}$$

where f und g are arbitrary functions of one variable.