QFT II - Problem Set 2

(29) Cross sections for simple cases

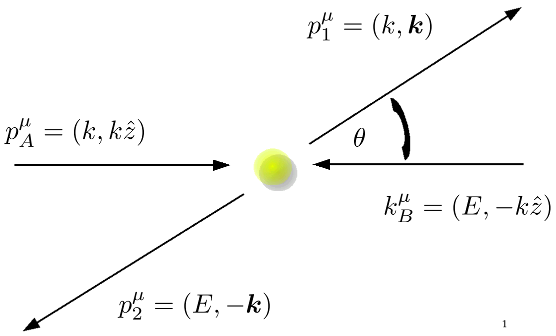
The cross section for two particles in the final state is given by

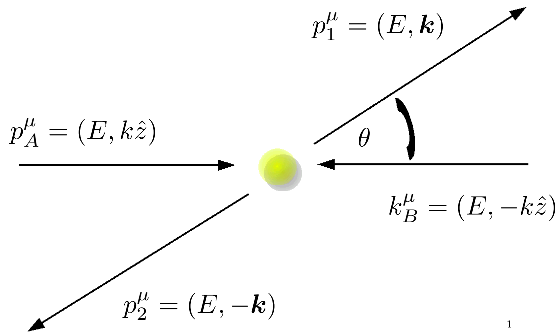
$$\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{1}{2E_A 2E_B |v_A - v_B|} \frac{|\boldsymbol{p_1}|}{(2\pi)^2 4E_{cm}} |\mathcal{M}(p_A, p_B \to p_1, p_2)|^2,$$

where the incoming particles are labeled A, B, the outgoing particles p_1, p_2 and E_{cm} is the center of mass energy.

a) Suppose that one of them (particle A) is massless, like in the left figure below. Denote $p_A^\mu = (k, k\hat{z})$, $p_B^\mu = (E, -k\hat{z})$. What is the cross section in this case? *Hint: use* $\boldsymbol{p} = m\boldsymbol{v}/\sqrt{1-v^2}$ *to get an expression for* v_B *in terms of* $\boldsymbol{p}_B = -k\hat{z}$ *and* $E_B = E$.

b) Suppose that all particles have the same mass, like in the right figure below. What is the cross section then?





(30) Feynman rules for everyone

In general, you can derive Feynman rules for any theory by simply taking derivatives of the action S with respect to the fields. The momentum conserving δ functions that occur in the derivatives are omitted in the expressions for the propagator and the vertex. They do, however yield the rules for momentum conservation at each vertex.

Drawing a Propagator in a Feynam diagram is associated with

$$P = \frac{i}{\mathcal{S}^{(2)}},$$

where

$$\mathcal{S}^{(2)} = \frac{\delta^2 S(\phi, \chi, \dots)}{\delta\phi\delta\chi}_{|\phi=\chi=\dots=0}.$$

Likewise, the vertices are higher derivatives of S, again evaluated at vanishing field values. For instance the vertex for the interaction Lagrangian $\mathcal{L}(\phi, \chi) = \lambda\chi\chi\phi\phi/4$ is

$$V = i\frac{\delta^4 S(\phi, \chi, \dots)}{\delta^2\phi\delta^2\chi}_{|\phi=\chi=\dots=0} = i\lambda$$

a) To flex our muscles a bit, let's apply this for several theories. So what are the propagator(s) and vertex (vertices) for the following Lagrangians ?

i) Scalar ϕ^4 theory

$$\mathcal{S} = -\int_p \frac{1}{2}(p_\mu p^\mu + m^2)\phi(p)\phi(-p) - \frac{\lambda}{4!}\int_{p_1 \dots p_4} \delta(p_1 + p_2 + p_3 + p_4)\phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4)$$

ii) Complex scalar ϕ^4 theory

$$\mathcal{S} = -\int_p (p_\mu p^\mu + m^2)\phi^*(p)\phi(p) - \frac{\lambda}{4}\int_{p_1 \dots p_4} \delta(p_1 - p_2 + p_3 - p_4)\phi(p_1)\phi^*(p_2)\phi(p_3)\phi^*(p_4)$$

b) The interaction of a dipole with the electric field is described by the following action

$$S_{int} = -\int_x \frac{i}{2}\bar{\Psi}(x)\sigma^{\mu\nu}(M + \gamma^5 D)\Psi(x)F_{\mu\nu}(x),$$

where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu]$ is the usual spin operator and M and D are the magnetic and electric dipole strength.

(i) Fourier transform S_{int} to momentum space (please use the letter k for the momentum of A_μ).

(ii) $\sigma_{\mu\nu}$ is antisymmetric in μ, ν. Use this to add a "clever" 0 to $F_{\mu\nu}$ to show that you can simplify $\sigma^{\mu\nu}F_{\mu\nu} \to 2\sigma^{\mu\nu}k_\mu A_\nu$, where k is the photon momentum.

(iii) What is the vertex for this interaction ? Draw it. What happens to the vertex for very soft photons ?

(iv) Draw the Feynman diagramm for the scattering of an electron and a dark matter fermion that carries an electric dipole moment D but no charge.

(v) Obtain the amplitude $\mathcal{M}$ for this process.

(31) Bhabha scattering

Let us consider electron-positron (Bhaba) scattering. At tree level, there are two diagrams contributing. One in which electrons and positrons simply scatter and one where they annihilate and re-appear later.

a) Draw the Feynman diagrams for both processes.

b) Use the QED Feynman rules to obtain the amplitude $\mathcal{M}$ and $|\mathcal{M}|^2$.

c) Average over incoming spins and sum over outgoing spins, i.e. use the spin sum rules to obtain $|\mathcal{M}|^2$ in terms of Traces.

d) Stop here. Don't evaluate the traces ...